## Student Notes and Problems

PHYSICS 11 British Columbia

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## INTRODUCTION TO PHYSICS

Lesson 2—Significant Digits and Scientific Notation

PRACTICE EXERCISES ANSWERS AND SOLUTIONS

1. a) 3
b) 2
c) 3
d) 1
e) 3
2. a) $3.40 \times 10^{-3} \mathrm{~m}$
b) $7.0 \times 10^{2} \mathrm{~m}$
c) $5.6 \times 10^{2} \mathrm{~m}$
d) $5.5 \times 10^{2} \mathrm{~m}$
e) $4.1 \times 10^{2} \mathrm{~m}$
3. a) $7.23 \times 10^{1} \mathrm{~m}$
b) $5.59 \times 10^{2} \mathrm{~m}$
c) $4.90 \times 10^{2} \mathrm{~m}$
4. a) $6.2 \times 10^{1} \mathrm{~m}^{2}$
b) $1.8 \times 10^{1} \mathrm{~m}^{2}$
c) $1.24 \times 10^{4} \mathrm{~m}^{2}$

## Lesson 3-Graphing

## PRACTICE EXERCISES

 ANSWERS AND SOLUTIONS1. a)

b) slope $=\frac{\text { rise }}{\text { run }}=\frac{y_{2}-y_{1}}{x_{2}-x_{1}}$

$$
\begin{aligned}
& =\frac{0.750 \mathrm{eb}-0}{15.0 \mathrm{tz} / \mathrm{s}-0} \\
& =5.00 \times 10^{-2} \mathrm{eb} \cdot \mathrm{~s} / \mathrm{tz}
\end{aligned}
$$

c) $A \propto B$ or $A=k B$
2. a)

b)

| $\boldsymbol{B}\left(\times \mathbf{1 0}^{-\mathbf{2}} \mathbf{t z} / \mathbf{s}\right)$ | $\boldsymbol{A}(\mathbf{e b})$ | $\frac{\mathbf{1}}{\boldsymbol{B}}(\mathbf{s} / \mathbf{t z})$ |
| :---: | :---: | :--- |
| 2.1 | 4.0 | 47.6 |
| 1.4 | 6.0 | 71.4 |
| 1.1 | 8.0 | 90.9 |
| 0.7 | 12.0 | 143 |
| 0.35 | 24.0 | 286 |


c) slope $=\frac{\text { rise }}{\text { run }}=\frac{y_{2}-y_{1}}{x_{2}-x_{1}}$

$$
\begin{aligned}
& =\frac{24 \mathrm{eb}-0}{286 \mathrm{~s} / \mathrm{tz}-0} \\
& =8.40 \times 10^{-2} \mathrm{eb} \cdot \mathrm{tz} / \mathrm{s}
\end{aligned}
$$

d) $A \propto \frac{1}{B}$ or $A=\frac{k}{B}$
3. a)

b)

| $\boldsymbol{B}(\mathrm{tz} / \mathbf{s})$ | $\boldsymbol{A}(\mathbf{e b})$ | $\left.\boldsymbol{B}^{\mathbf{2}} \mathbf{( t z}^{\mathbf{2}} \mathbf{s}^{\mathbf{2}}\right)$ |
| :---: | :---: | :---: |
| 1.5 | 2.25 | 2.25 |
| 3.0 | 9.00 | 9.00 |
| 4.5 | 20.25 | 20.25 |
| 6.0 | 36.00 | 36.00 |
| 7.5 | 56.25 | 56.25 |
| 9.0 | 81.00 | 81.00 |


c) slope $=\frac{\text { rise }}{\text { run }}=\frac{y_{2}-y_{1}}{x_{2}-x_{1}}$

$$
\begin{aligned}
& =\frac{70 \mathrm{eb}-0}{70 \mathrm{tz}^{2} / \mathrm{s}^{2}-0} \\
& =1.00 \mathrm{eb} \cdot \mathrm{~s}^{2} / \mathrm{tz}^{2}
\end{aligned}
$$

d) $A \propto B^{2}$ or $A=\left(1.00 \mathrm{eb} \cdot \mathrm{s}^{2} / \mathrm{tz}^{2}\right) B^{2}$

## MOTION

## Lesson 1-Introduction to Kinematics

## PRACTICE EXERCISES ANSWERS AND SOLUTIONS

1. a) $d=275 \mathrm{~m}+425 \mathrm{~m}$

$$
=700 \mathrm{~m}
$$

b) $\quad \vec{d}=275 \mathrm{~m}+(-425 \mathrm{~m})$

$$
=-150 \mathrm{~m} \text { or } 150 \mathrm{~m} \text { left }
$$

2. a) $d=2(115 \mathrm{~m})+2(125 \mathrm{~m})$

$$
=480 \mathrm{~m}
$$

b) Vertical direction

$$
\begin{aligned}
\vec{d} & =125 \mathrm{~m}+(-125 \mathrm{~m}) \\
& =0 \mathrm{~m}
\end{aligned}
$$

Horizontal direction
$\vec{d}=115 \mathrm{~m}+(-115 \mathrm{~m})$

$$
=0 \mathrm{~m}
$$

$\therefore \vec{d}=0$
$3 \quad$ a) $\quad v_{\mathrm{ave}}=\frac{d}{t}$

$$
\begin{aligned}
v_{\mathrm{ave}} & =\frac{(11 \mathrm{~m}+25 \mathrm{~m})}{52 \mathrm{~m}} \\
& =0.69 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

b) $\quad \vec{v}_{\text {ave }}=\frac{\vec{d}}{t}$

$$
\begin{aligned}
\vec{v}_{\mathrm{ave}} & =\frac{(11 \mathrm{~m}+(-25 \mathrm{~m}))}{52 \mathrm{~s}} \\
& =-0.27 \mathrm{~m} / \mathrm{s} \text { or } 0.27 \mathrm{~m} / \mathrm{s} \text { left }
\end{aligned}
$$

## Lesson 2-Instantaneous Velocity and Speed

## PRACTICE EXERCISES

 ANSWERS AND SOLUTIONS1. $\vec{v}_{\mathrm{ave}}=\frac{\vec{d}}{t}$

$$
\begin{aligned}
& =\frac{1.00 \times 10^{2} \mathrm{~m}}{11.2 \mathrm{~s}} \\
& =8.93 \mathrm{~m} / \mathrm{s} \mathrm{right}
\end{aligned}
$$

2. a) $v=\frac{d}{t}$
$d=v t$

$$
\begin{aligned}
& =(10.0 \mathrm{~m} / \mathrm{s})(4.5 \mathrm{~s}) \\
& =45 \mathrm{~m}
\end{aligned}
$$

b) $\vec{v}=\frac{\vec{d}}{t}$

$$
\vec{d}=\vec{v} t
$$

$$
=(-10.0 \mathrm{~m} / \mathrm{s})(4.5)
$$

$=-45 \mathrm{~m}$ or 45 m left
3. $v=\frac{d}{t}$

$$
t=\frac{d}{v}
$$

$$
=\frac{2.5 \mathrm{~m}}{9.8 \mathrm{~m} / \mathrm{s}}
$$

$$
=0.26 \mathrm{~s}
$$

4. 1st part of the motion:

$$
\begin{aligned}
\stackrel{\rightharpoonup}{v}_{\text {ave }} & =\frac{\vec{d}}{t} \\
\vec{d} & =\vec{v}_{\text {ave }} t \\
& =(1.30 \mathrm{~m} / \mathrm{s})(98.0 \mathrm{~s}) \\
& =127.4 \mathrm{~m} \text { right }
\end{aligned}
$$

2nd part of the motion:

$$
\begin{aligned}
\vec{d} & =\stackrel{\rightharpoonup}{\text { ave }} t \\
& =(0.45 \mathrm{~m} / \mathrm{s})(90.0 \mathrm{~s}) \\
& =40.5 \mathrm{~m} \text { right }
\end{aligned}
$$

Total displacement:

$$
\begin{aligned}
\vec{d} & =40.5 \mathrm{~m}+127.4 \mathrm{~m} \\
& =167.9 \mathrm{~m} \text { right }
\end{aligned}
$$

$$
\vec{v}_{\mathrm{ave}}=\frac{\vec{d}}{t}
$$

$$
=\frac{167.9 \mathrm{~m}}{188 \mathrm{~s}}
$$

$$
=0.893 \mathrm{~m} / \mathrm{s} \text { right }
$$

5. $\quad \vec{v}_{\text {ave }}=\frac{\vec{d}}{t}$

$$
\begin{aligned}
& =\frac{-64.0 \mathrm{~m}}{3.61 \mathrm{~s}} \\
& =-17.7 \mathrm{~m} / \mathrm{s} \text { or } 17.7 \mathrm{~m} / \mathrm{s} \text { down }
\end{aligned}
$$

6. Velocity is the slope of the displacement-time graph.

$$
\begin{aligned}
\text { slope } & =\frac{\text { rise }}{\text { run }} \\
& =\frac{(10.0-0) \mathrm{m}}{(15.0-0) \mathrm{s}} \\
\vec{v} & =0.667 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

Because the slope is constant, the velocity is constant at any time $t$.
a) $0.667 \mathrm{~m} / \mathrm{s}$ right
b) $0.667 \mathrm{~m} / \mathrm{s}$ right
c) $0.667 \mathrm{~m} / \mathrm{s}$ right
7. Both speed and velocity are equal to the slope of the graph.

$$
\begin{aligned}
\text { slope } & =\frac{\text { rise }}{\text { run }} \\
& =\frac{(8.0-0) \mathrm{m}}{(8.0-0) \mathrm{s}} \\
& =1.0 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

a) $v=1.0 \mathrm{~m} / \mathrm{s}$
b) $\quad \vec{v}=1.0 \mathrm{~m} / \mathrm{s}$ right
8. Both the displacement and distance are found by determining the area under the graph.

$$
\text { area }=l \times w
$$

$$
\begin{aligned}
& =(10.0 \mathrm{~s})(7.0 \mathrm{~m} / \mathrm{s}) \\
& =7.0 \times 10^{1} \mathrm{~m}
\end{aligned}
$$

a) $\vec{d}=7.0 \times 10^{1} \mathrm{~m}$ left
b) $d=7.0 \times 10^{1} \mathrm{~m}$
9. Both the displacement and distance are found by determining the area under the graph.
area $=l \times w$

$$
\begin{aligned}
& =(5.0 \mathrm{~s})(2.5 \mathrm{~m}) \\
& =13 \mathrm{~m}
\end{aligned}
$$

a) $\vec{d}=13 \mathrm{~m}$ left
b) $d=13 \mathrm{~m}$
10. 1st part of the motion:

$$
\begin{aligned}
\stackrel{\rightharpoonup}{v} & =\frac{\vec{d}}{t} \\
\vec{d} & =\stackrel{\rightharpoonup}{v} t \\
& =(5.0 \mathrm{~m} / \mathrm{s})(31 \mathrm{~min} \times 60 \mathrm{~s} / \mathrm{min}) \\
& =9.3 \times 10^{3} \mathrm{~m} \text { right }
\end{aligned}
$$

2nd part of the motion:
$\vec{d}=\stackrel{\rightharpoonup}{v} t$
$=(7.0 \mathrm{~m} / \mathrm{s})(15 \mathrm{~min} \times 60 \mathrm{~s} / \mathrm{min})$
$=6.3 \times 10^{3} \mathrm{~m}$ right

Total displacement:

$$
\begin{aligned}
\vec{d} & =9.3 \times 10^{3} \mathrm{~m}+6.3 \times 10^{3} \mathrm{~m} \\
& =1.56 \times 10^{4} \mathrm{~m} \mathrm{right} \\
\vec{v}_{\text {ave }} & =\frac{\vec{d}}{t} \\
& =\frac{1.56 \times 10^{4} \mathrm{~m}}{46 \mathrm{~min} \times 60 \mathrm{~s} / \mathrm{min}} \\
& =5.7 \mathrm{~m} / \mathrm{s} \mathrm{right}
\end{aligned}
$$

11. 1st part of the motion:

$$
\begin{aligned}
& \stackrel{\rightharpoonup}{v}=\frac{\vec{d}}{t} \\
& \vec{d}=\stackrel{\rightharpoonup}{v} t \\
&=(8.0 \mathrm{~m} / \mathrm{s})(25 \mathrm{~min} \times 60 \mathrm{~s} / \mathrm{min}) \\
&=1.20 \times 10^{4} \mathrm{~m} \text { right } \\
& \text { 2nd part of the motion: } \\
& \begin{aligned}
\vec{d}_{\mathrm{av}} & =\vec{v} t \\
& =(-5.0 \mathrm{~m} / \mathrm{s})(15 \mathrm{~min} \times 60 \mathrm{~s} / \mathrm{min}) \\
& =-4.50 \times 10^{3} \mathrm{~m} \text { or } 4.50 \times 10^{3} \mathrm{~m} \text { left }
\end{aligned}
\end{aligned}
$$

Total displacement:

$$
\begin{aligned}
\vec{d} & =1.20 \times 10^{4} \mathrm{~m}-4.50 \times 10^{3} \mathrm{~m} \\
& =7.5 \times 10^{3} \mathrm{~m} \text { right }
\end{aligned}
$$

Total distance:

$$
\begin{aligned}
d & =1.20 \times 10^{4} \mathrm{~m}+4.50 \times 10^{3} \mathrm{~m} \\
& =1.65 \times 10^{4} \mathrm{~m}
\end{aligned}
$$

a) $\quad \vec{v}_{\mathrm{ave}}=\frac{\vec{d}}{t}$

$$
\begin{aligned}
& =\frac{7.5 \times 10^{3} \mathrm{~m}}{40 \mathrm{~min} \times 60 \mathrm{~s} / \mathrm{min}} \\
& =3.1 \mathrm{~m} / \mathrm{s} \text { right }
\end{aligned}
$$

b) $\quad v_{\text {ave }}=\frac{d}{t}$

$$
\begin{aligned}
& =\frac{1.65 \times 10^{4} \mathrm{~m}}{40 \mathrm{~min} \times 60 \mathrm{~s} / \mathrm{min}} \\
& =6.9 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

12. let $t=$ time of motion of B
$\therefore t+60 \mathrm{~s}=$ time of motion of A

It is known that:
$\vec{d}_{\mathrm{A}}=\vec{d}_{\mathrm{B}}$
Find the displacement for Object B:

$$
\begin{aligned}
\vec{v}_{\mathrm{B}} & =\frac{\vec{d}_{\mathrm{B}}}{t} \\
3.0 \mathrm{~m} / \mathrm{s} & =\frac{\vec{d}_{\mathrm{B}}}{t} \\
\vec{d}_{\mathrm{B}} & =(3.0 \mathrm{~m} / \mathrm{s}) t
\end{aligned}
$$

Object A:

$$
\begin{aligned}
\vec{v}_{\mathrm{A}} & =\frac{\vec{d}_{\mathrm{A}}}{t} \quad \text { sub in } \vec{d}_{\mathrm{A}}=\vec{d}_{\mathrm{B}} \\
2.0 \mathrm{~m} / \mathrm{s} & =\frac{\vec{d}_{\mathrm{B}}}{t+60 \mathrm{~s}} \\
2.0 \mathrm{~m} / \mathrm{s} & =\frac{(3.0 \mathrm{~m} / \mathrm{s}) t}{t+60 \mathrm{~s}} \\
2.0(t+60 \mathrm{~s}) & =3.0 t \\
2.0 t+120 \mathrm{~s} & =3.0 t \\
120 \mathrm{~s} & =t \\
t & =1.2 \times 10^{2} \mathrm{~s}
\end{aligned}
$$

13. a)

| time <br> $(\mathrm{s})$ | dis placement <br> $(\mathrm{m})$ | dis placement <br> during time <br> interval <br> $(\mathrm{m})$ | average <br> velocity <br> during time <br> interval <br> $(\mathrm{m} / \mathrm{s})$ |
| :---: | :---: | :---: | :---: |
| 0 | 0 | 0.012 | 0.12 |
| 0.10 | 0.012 | 0.012 | 0.12 |
| 0.20 | 0.024 | 0.011 | 0.11 |
| 0.30 | 0.035 | 0.012 | 0.12 |
| 0.40 | 0.047 | 0.013 | 0.13 |
| 0.50 | 0.060 | 0.012 | 0.12 |
| 0.60 | 0.072 | 0.013 | 0.13 |
| 0.70 | 0.085 | 0.012 | 0.12 |
| 0.80 | 0.097 | 0.011 | 0.11 |
| 0.90 | 0.108 | 0.012 | 0.12 |
| 1.00 | 0.120 |  |  |

b) Position-time graph:

c) Velocity-Time Graph:

d) slope $=\frac{\text { rise }}{\text { run }}$

$$
\begin{aligned}
& =\frac{(0.12-0) \mathrm{m}}{(1.0-0) \mathrm{s}} \\
\vec{v}_{\text {ave }} & =0.12 \mathrm{~m} / \mathrm{s} \text { right }
\end{aligned}
$$

e) Displacement is the area under the velocity time graph.

$$
\begin{aligned}
\text { area } & =l \times w \\
& =(0.95 \mathrm{~s})(0.12 \mathrm{~m} / \mathrm{s}) \\
\vec{d} & =0.11 \mathrm{~m} \mathrm{right}
\end{aligned}
$$

14. a)

| time <br> $(\mathrm{s})$ | displacement <br> from $\mathrm{t}=0$ <br> $(\mathrm{~m})$ | displacement <br> during time <br> interval <br> $(\mathrm{m})$ | average <br> velocity <br> during time <br> interval <br> $(\mathrm{m} / \mathrm{s})$ |
| :---: | :---: | :---: | :---: |
| 0 | 0 | 0.016 | 0.16 |
| 0.10 | 0.016 | 0.015 | 0.15 |
| 0.20 | 0.031 | 0.015 | 0.15 |
| 0.30 | 0.046 | 0.016 | 0.16 |
| 0.40 | 0.062 | 0.014 | 0.14 |
| 0.50 | 0.076 | 0.015 | 0.15 |
| 0.60 | 0.091 |  |  |

b) Position-Time Graph:

c) Velocity-Time Graph:

d) i) Velocity is the slope of the position-time graph.

$$
\begin{aligned}
\text { slope } & =\frac{\text { rise }}{\text { run }} \\
& =\frac{(0.084-0) \mathrm{m}}{(0.55-0) \mathrm{s}} \\
\stackrel{\rightharpoonup}{v} & =0.15 \mathrm{~m} / \mathrm{s} \text { right }
\end{aligned}
$$

ii) Displacement is the area under the velocity-time graph.

$$
\begin{aligned}
\text { area } & =l \times w \\
& =(0.55 \mathrm{~s})(0.152 \mathrm{~m} / \mathrm{s}) \\
\vec{d} & =0.084 \mathrm{~m} \mathrm{right}
\end{aligned}
$$

## Lesson 3-Uniformly Accelerated <br> Motion in One Direction

## PRACTICE EXERCISES ANSWERS AND SOLUTIONS

1. a) Draw a tangent line at 0.40 s .

Find the slope of this tangent line.

$$
\begin{aligned}
\text { slope } & =\frac{\text { rise }}{\text { run }} \\
& =\frac{(5.0-1.5) \mathrm{m}}{(0.64-0.30) \mathrm{s}} \\
\vec{v} & =1.0 \times 10^{1} \mathrm{~m} / \mathrm{s} \text { right }
\end{aligned}
$$

b) Draw a tangent line at 0.60 s .

Find the slope of this tangent line.

$$
\begin{aligned}
\text { slope } & =\frac{\text { rise }}{\text { run }} \\
& =\frac{(9.0-2.0) \mathrm{m}}{(0.90-0.38) \mathrm{s}} \\
\vec{v} & =13 \mathrm{~m} / \mathrm{s} \text { right }
\end{aligned}
$$

2. a) Acceleration is the slope of the velocity-time graph.

$$
\begin{aligned}
\text { slope } & =\frac{\text { rise }}{\text { run }} \\
& =\frac{(15.0-0) \mathrm{m} / \mathrm{s}}{(7.0-0) \mathrm{s}} \\
\vec{a} & =2.1 \mathrm{~m} / \mathrm{s}^{2} \text { right }
\end{aligned}
$$

b) Displacement is the area under the velocity-time graph.

$$
\begin{aligned}
\text { area } & =\frac{1}{2}(l \times w) \\
& =\frac{1}{2}(7.0 \mathrm{~s} \times 15.0 \mathrm{~m} / \mathrm{s}) \\
\vec{d} & =53 \mathrm{~m} \text { right }
\end{aligned}
$$

3. a) Acceleration is the slope of the velocity-time graph.

$$
\begin{aligned}
\text { slope } & =\frac{\text { rise }}{\text { run }} \\
& =\frac{(13.0-0) \mathrm{m} / \mathrm{s}}{(9.0-0) \mathrm{s}}
\end{aligned}
$$

$$
\vec{a}=1.4 \mathrm{~m} / \mathrm{s}^{2} \text { right }
$$

b) Displacement is the area under the velocitytime graph.

$$
\begin{aligned}
\text { area } & =\frac{1}{2}(l \times w) \\
& =\frac{1}{2}(10.0 \mathrm{~s} \times 14.5 \mathrm{~m} / \mathrm{s}) \\
\vec{d} & =73 \mathrm{~m} \text { right }
\end{aligned}
$$

4. a) $\vec{v}_{\mathrm{A}}=9.0 \mathrm{~m} / \mathrm{s}$ right
$\vec{v}_{\mathrm{B}}=4.5 \mathrm{~m} / \mathrm{s}$ right
ratio $=\frac{\vec{v}_{\mathrm{A}}}{\vec{v}_{\mathrm{B}}}$

$$
=\frac{9.0 \mathrm{~m} / \mathrm{s}}{4.5 \mathrm{~m} / \mathrm{s}}
$$

$$
=2.0
$$

A is travelling 2.0 times faster than B .
b) Distance is the area under the velocity-time graph.
area $=\frac{1}{2}(l \times w)$
Distance A travels:

$$
\begin{aligned}
\text { area } & =\frac{1}{2}(5.0 \mathrm{~s} \times 9.0 \mathrm{~m} / \mathrm{s}) \\
d_{\mathrm{A}} & =22.5 \mathrm{~m}
\end{aligned}
$$

Distance B travels:

$$
\begin{aligned}
& \text { area }=\frac{1}{2}(5.0 \mathrm{~s} \times 4.5 \mathrm{~m} / \mathrm{s}) \\
& d_{\mathrm{B}}=11.3 \mathrm{~m} \\
& \text { difference }=22.5 \mathrm{~m}-11.3 \mathrm{~m} \\
& =11.2 \mathrm{~m}
\end{aligned}
$$

5. a) Acceleration is the slope of the velocity-time graph.

$$
\begin{aligned}
\text { slope } & =\frac{\text { rise }}{\text { run }} \\
& =\frac{(16.0-0) \mathrm{m} / \mathrm{s}}{(10.0-0) \mathrm{s}} \\
\vec{a} & =1.60 \mathrm{~m} / \mathrm{s}^{2} \mathrm{left}
\end{aligned}
$$

b) Displacement is the area under the velocitytime graph.

$$
\begin{aligned}
\text { area } & =\frac{1}{2}(l \times w) \\
& =\frac{1}{2}(10.0 \mathrm{~s} \times 16.0 \mathrm{~m} / \mathrm{s}) \\
\vec{d} & =80.0 \mathrm{~m} \mathrm{left}
\end{aligned}
$$

6. a) Acceleration is the slope of the velocity-time graph.

$$
\begin{aligned}
\text { slope } & =\frac{\text { rise }}{\text { run }} \\
& =\frac{(10.0-4.0) \mathrm{m} / \mathrm{s}}{(16.0-0) \mathrm{s}} \\
\vec{a} & =0.38 \mathrm{~m} / \mathrm{s}^{2} \mathrm{left}
\end{aligned}
$$

b) Displacement is the area under the velocitytime graph.

$$
\begin{aligned}
\text { area } & =(l \times w)+\frac{1}{2}(l \times w) \\
& =(16.0 \mathrm{~s} \times 4.0 \mathrm{~m} / \mathrm{s})+\frac{1}{2}(16.0 \mathrm{~s} \times 6.0 \mathrm{~m} / \mathrm{s}) \\
\vec{d} & =1.1 \times 10^{2} \mathrm{~m} \text { left }
\end{aligned}
$$

7. a) Acceleration is the slope of the velocity-time graph.

$$
\begin{aligned}
\text { slope } & =\frac{\text { rise }}{\text { run }} \\
& =\frac{(32.5-10.0) \mathrm{m} / \mathrm{s}}{(10.0-0) \mathrm{s}} \\
\vec{a} & =2.25 \mathrm{~m} / \mathrm{s}^{2} \mathrm{right}
\end{aligned}
$$

b) Displacement is the area under the velocitytime graph.
area $=(l \times w)+\frac{1}{2}(l \times w)$
$=(10.0 \mathrm{~s} \times 10.0 \mathrm{~m} / \mathrm{s})+\frac{1}{2}(10.0 \mathrm{~s} \times 22.5 \mathrm{~m} / \mathrm{s})$
$\vec{d}=213 \mathrm{~m}$ right
8. a) Velocity-Time Graph

b) i) Acceleration is the slope of the velocity-time graph.

$$
\begin{aligned}
\text { slope } & =\frac{\text { rise }}{\text { run }} \\
& =\frac{(31.8-12.0) \mathrm{m} / \mathrm{s}}{(6.0-0.0) \mathrm{s}} \\
\vec{a} & =3.3 \mathrm{~m} / \mathrm{s}^{2} \text { right }
\end{aligned}
$$

ii) Displacement is the area under the velocity-time graph.

$$
\begin{aligned}
& \text { area }=(l \times w)+\frac{1}{2}(l \times w) \\
& =(6.0 \mathrm{~s} \times 12.0 \mathrm{~m} / \mathrm{s})+\frac{1}{2}(6.0 \mathrm{~s} \times 19.8 \mathrm{~m} / \mathrm{s}) \\
& \vec{d}=1.3 \times 10^{2} \mathrm{~m} \text { right }
\end{aligned}
$$

9. a)
$\left.\begin{array}{|c|c|c|c|}\hline & & & \begin{array}{c}\text { Average } \\ \text { velocity } \\ \text { during } \\ \text { (s) }\end{array} \\ \hline 0 & 0 & \begin{array}{c}\text { Displacement } \\ \text { from } t=0(\mathbf{m})\end{array} & \begin{array}{c}\text { Displacement } \\ \text { during time } \\ \text { interval }(\mathbf{m})\end{array}\end{array} \begin{array}{c}\text { interval } \\ (\mathbf{m} / \mathbf{s})\end{array}\right]$
b) Position-Time Graph

c) Velocity-Time Graph

d) Acceleration is the slope of the velocity-time graph.

$$
\begin{aligned}
\text { slope } & =\frac{\text { rise }}{\text { run }} \\
& =\frac{(0.22-0) \mathrm{m} / \mathrm{s}}{(0.45-0) \mathrm{s}}
\end{aligned}
$$

$$
\vec{a}=0.49 \mathrm{~m} / \mathrm{s}^{2} \text { right }
$$

10. a)

|  |  |  | Average <br> velocity <br> during <br> (s) |
| :---: | :---: | :---: | :---: |
| 0 | 0.0 | Displacement <br> from $\boldsymbol{t}=\mathbf{0}(\mathbf{m})$ | Displacement <br> during time <br> interval (m) |
| interval |  |  |  |
| $(\mathbf{m} / \mathbf{s})$ |  |  |  |$|$

b) Position-Time Graph

c) Velocity-Time Graph

d) Acceleration is the slope of the velocity-time graph.

$$
\begin{aligned}
\text { slope } & =\frac{\text { rise }}{\text { run }} \\
& =\frac{(2.25-0) \mathrm{m} / \mathrm{s}}{(0.50-0) \mathrm{s}} \\
\vec{a} & =4.5 \mathrm{~m} / \mathrm{s}^{2} \text { right }
\end{aligned}
$$

Acceleration-Time Graph

e) To find the velocity, read your velocity-time graph.

At $0.30 \mathrm{~s}, \vec{v}=1.3 \mathrm{~m} / \mathrm{s}$ right
At $0.60 \mathrm{~s}, \vec{v}=2.7 \mathrm{~m} / \mathrm{s}$ right

Note: in the following questions, consider right as the positive direction and left as the negative direction.
11.

| $\vec{v}_{\mathrm{i}}$ | $\vec{v}_{\mathrm{f}}$ | $\vec{a}$ | $\vec{d}$ | $t$ |
| :---: | :---: | :---: | :---: | :---: |
| 0 | $-12.0 \mathrm{~m} / \mathrm{s}$ | $?$ | $\times$ | 3.40 s |

$$
\begin{aligned}
\vec{a} & =\frac{\bar{v}_{\mathrm{f}}-\bar{v}_{\mathrm{i}}}{t} \\
& =\frac{(-12.0 \mathrm{~m} / \mathrm{s})-0}{3.40 \mathrm{~s}} \\
& =-3.53 \mathrm{~m} / \mathrm{s}^{2} \text { or } 3.53 \mathrm{~m} / \mathrm{s}^{2} \mathrm{left}
\end{aligned}
$$

12. 

| $\vec{v}_{\mathrm{i}}$ | $\vec{v}_{\mathrm{f}}$ | $\vec{a}$ | $\vec{d}$ | $t$ |
| :---: | :---: | :---: | :---: | :---: |
| 0 | $15 \mathrm{~m} / \mathrm{s}$ | $\times$ | $?$ | 4.7 s |

$$
\begin{aligned}
\vec{d} & =\left(\frac{\stackrel{\rightharpoonup}{\mathrm{f}}_{\mathrm{f}}+\vec{v}_{\mathrm{i}}}{2}\right) t \\
& =\left(\frac{15 \mathrm{~m} / \mathrm{s}+0}{2}\right) 4.7 \mathrm{~s} \\
& =35 \mathrm{~m} \text { right }
\end{aligned}
$$

13. 

| $\vec{v}_{\mathrm{i}}$ | $\vec{v}_{\mathrm{f}}$ | $\vec{a}$ | $\vec{d}$ | $t$ |
| :---: | :---: | :---: | :---: | :---: |
| 0 | $?$ | $1.9 \mathrm{~m} / \mathrm{s}^{2}$ | $?$ | 5.0 s |

a) $\vec{d}=\vec{v}_{\mathrm{i}} t+\frac{1}{2} \vec{a} t^{2}$

$$
\begin{aligned}
& =\frac{1}{2}\left(1.9 \mathrm{~m} / \mathrm{s}^{2}\right)(5.0 \mathrm{~s})^{2} \\
& =24 \mathrm{~m} \text { right }
\end{aligned}
$$

b) $\quad \vec{a}=\frac{\vec{v}_{\mathrm{f}}-\vec{v}_{\mathrm{i}}}{t}$

$$
1.9 \mathrm{~m} / \mathrm{s}^{2}=\frac{\vec{v}_{\mathrm{f}}-0}{5.0 \mathrm{~s}}
$$

$$
\vec{v}_{\mathrm{f}}=9.5 \mathrm{~m} / \mathrm{s} \text { right }
$$

c) The magnitude of the displacement (distance) is 24 m .
d) The magnitude of velocity (speed) is $9.5 \mathrm{~m} / \mathrm{s}$.
14.

| $\vec{v}_{\mathrm{i}}$ | $\vec{v}_{\mathrm{f}}$ | $\vec{a}$ | $\vec{d}$ | $t$ |
| :---: | :---: | :---: | :---: | :---: |
| $-2.0 \mathrm{~m} / \mathrm{s}$ | $?$ | $-1.3 \mathrm{~m} / \mathrm{s}^{2}$ | -15 m | $\times$ |

a) $\quad \vec{v}_{\mathrm{f}}^{2}=\vec{v}_{\mathrm{i}}^{2}+2 \vec{a} \vec{d}$

$$
\begin{aligned}
& =(-2.0 \mathrm{~m} / \mathrm{s})^{2}+2\left(-1.3 \mathrm{~m} / \mathrm{s}^{2}\right)(-15 \mathrm{~m}) \\
\stackrel{v}{\mathrm{f}} & =-6.6 \mathrm{~m} / \mathrm{s} \text { or } 6.6 \mathrm{~m} / \mathrm{s} \text { left }
\end{aligned}
$$

b) The magnitude of velocity (speed) is $6.6 \mathrm{~m} / \mathrm{s}$.
15.

| $\vec{v}_{\mathrm{i}}$ | $\vec{v}_{\mathrm{f}}$ | $\vec{a}$ | $\vec{d}$ | $t$ |
| :---: | :---: | :---: | :---: | :---: |
| $5.0 \mathrm{~m} / \mathrm{s}$ | $?$ | $3.0 \mathrm{~m} / \mathrm{s}^{2}$ | $\times$ | 2.9 s |

$$
\begin{aligned}
\vec{a} & =\frac{\vec{v}_{\mathrm{f}}-\vec{v}_{\mathrm{i}}}{t} \\
3.0 \mathrm{~m} / \mathrm{s}^{2} & =\frac{\vec{v}_{\mathrm{f}}-5.0 \mathrm{~m} / \mathrm{s}}{2.9 \mathrm{~s}} \\
\vec{v}_{\mathrm{f}} & =\left(3.0 \mathrm{~m} / \mathrm{s}^{2}\right)(2.9 \mathrm{~s})+5.0 \mathrm{~m} / \mathrm{s} \\
& =14 \mathrm{~m} / \mathrm{s} \text { right }
\end{aligned}
$$

16. 

| $\vec{v}_{\mathrm{i}}$ | $\vec{v}_{\mathrm{f}}$ | $\vec{a}$ | $\vec{d}$ | $t$ |
| :---: | :---: | :---: | :---: | :---: |
| 0 | $-11.0 \mathrm{~m} / \mathrm{s}$ | $\times$ | -26.0 m | $?$ |

$$
\begin{aligned}
\vec{d} & =\left(\frac{\vec{v}_{\mathrm{f}}+\vec{v}_{\mathrm{i}}}{2}\right) t \\
-26.0 \mathrm{~m} & =\left(\frac{-11.0 \mathrm{~m} / \mathrm{s}+0}{2}\right) t \\
t & =4.73 \mathrm{~s}
\end{aligned}
$$

17. 

| $\vec{v}_{\mathrm{i}}$ | $\vec{v}_{\mathrm{f}}$ | $\vec{a}$ | $\vec{d}$ | $t$ |
| :---: | :---: | :---: | :---: | :---: |
| 0 | $\times$ | $?$ | 20.0 m | 8.10 s |

$$
\begin{aligned}
\vec{d} & =\vec{v}_{\mathrm{i}} t+\frac{1}{2} \vec{a} t^{2} \\
20.0 \mathrm{~m} & =\frac{1}{2} \vec{a}(8.10 \mathrm{~s})^{2} \\
\vec{a} & =0.610 \mathrm{~m} / \mathrm{s}^{2} \text { right }
\end{aligned}
$$

18. 

| $\vec{v}_{\mathrm{i}}$ | $\vec{v}_{\mathrm{f}}$ | $\vec{a}$ | $\vec{d}$ | $t$ |
| :---: | :---: | :---: | :---: | :---: |
| $15 \mathrm{~km} / \mathrm{h}$ | $65 \mathrm{~km} / \mathrm{h}$ | $4.0 \mathrm{~m} / \mathrm{s}^{2}$ | $?$ | $\times$ |

$$
\begin{aligned}
\vec{v}_{i} & =15 \mathrm{~km} / \mathrm{h} \times 1000 \mathrm{~m} / \mathrm{km} \times \frac{1 \mathrm{~h}}{3600 \mathrm{~s}} \\
& =4.17 \mathrm{~m} / \mathrm{s} \\
\vec{v}_{\mathrm{f}} & =65 \mathrm{~km} / \mathrm{h} \times 1000 \mathrm{~m} / \mathrm{km} \times \frac{1 \mathrm{~h}}{3600 \mathrm{~s}} \\
& =18.1 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

$$
\begin{aligned}
\vec{v}_{\mathrm{f}}^{2} & =\bar{v}_{\mathrm{i}}^{2}+2 \vec{a} \vec{d} \\
(18.1 \mathrm{~m} / \mathrm{s})^{2} & =(4.17 \mathrm{~m} / \mathrm{s})^{2}+2\left(4.0 \mathrm{~m} / \mathrm{s}^{2}\right) \vec{d} \\
\vec{d} & =\frac{(18.1 \mathrm{~m} / \mathrm{s})^{2}-(4.17 \mathrm{~m} / \mathrm{s})^{2}}{2\left(4.0 \mathrm{~m} / \mathrm{s}^{2}\right)} \\
& =39 \mathrm{~m} \text { right }
\end{aligned}
$$

19. 

| $\vec{v}_{\mathrm{i}}$ | $\vec{v}_{\mathrm{f}}$ | $\vec{a}$ | $\vec{d}$ | $t$ |
| :---: | :---: | :---: | :---: | :---: |
| 0 | $122.0 \mathrm{~km} / \mathrm{h}$ | $\times$ | $\times$ | 10.5 s |

$$
\begin{aligned}
\stackrel{\rightharpoonup}{v a v e} & =\frac{\vec{v}_{\mathrm{f}}+\vec{v}_{\mathrm{i}}}{2} \\
& =\frac{122.0 \mathrm{~km} / \mathrm{h}+0}{2} \\
& =61.0 \mathrm{~km} / \mathrm{h} \\
& =61.0 \mathrm{~km} / \mathrm{h} \times 1000 \mathrm{~m} / \mathrm{km} \times \frac{1 \mathrm{~h}}{3600 \mathrm{~s}} \\
& =16.9 \mathrm{~m} / \mathrm{s} \text { right }
\end{aligned}
$$

20. 

| $\vec{v}_{\mathrm{i}}$ | $\vec{v}_{\mathrm{f}}$ | $\vec{a}$ | $\vec{d}$ | $t$ |
| :---: | :---: | :---: | :---: | :---: |
| 0 | $-12.0 \mathrm{~m} / \mathrm{s}$ | $-2.40 \mathrm{~m} / \mathrm{s}^{2}$ | $\times$ | $?$ |

$$
\begin{aligned}
\vec{a}_{\mathrm{ave}} & =\frac{\vec{v}_{\mathrm{f}}-\vec{v}_{\mathrm{i}}}{t} \\
-2.40 \mathrm{~m} / \mathrm{s}^{2} & =\frac{(-12.0 \mathrm{~m} / \mathrm{s})-0}{t} \\
t & =\frac{(-12.0 \mathrm{~m} / \mathrm{s})-0}{-2.40 \mathrm{~m} / \mathrm{s}^{2}} \\
& =5.00 \mathrm{~s}
\end{aligned}
$$

21. 

| $\vec{v}_{\mathrm{i}}$ | $\vec{v}_{\mathrm{f}}$ | $\vec{a}$ | $\vec{d}$ | $t$ |
| :---: | :---: | :---: | :---: | :---: |
| 0 | $?$ | $\times$ | 19.0 m | 7.10 s |

$$
\vec{d}=\left(\frac{\stackrel{\rightharpoonup}{\mathrm{f}}+\vec{v}_{\mathrm{i}}}{2}\right) t
$$

$19.0 \mathrm{~m}=\left(\frac{\overrightarrow{\mathrm{v}}_{\mathrm{f}}+0}{2}\right) 7.10 \mathrm{~s}$

$$
\begin{aligned}
\vec{v}_{\mathrm{f}} & =\frac{2(19.0 \mathrm{~m})}{7.10 \mathrm{~s}} \\
& =5.35 \mathrm{~m} / \mathrm{s} \mathrm{right}
\end{aligned}
$$

22. 

| $\vec{v}_{\mathrm{i}}$ | $\vec{v}_{\mathrm{f}}$ | $\vec{a}$ | $\vec{d}$ | $t$ |
| :---: | :---: | :---: | :---: | :---: |
| $?$ | $\times$ | $-0.71 \mathrm{~m} / \mathrm{s}^{2}$ | -8.0 m | 3.2 s |

$$
\begin{aligned}
\vec{d} & =\vec{v}_{\mathrm{i}} t+\frac{1}{2} \vec{a} t^{2} \\
-8.0 \mathrm{~m} & =\vec{v}_{\mathrm{i}}(3.2 \mathrm{~s})+\frac{1}{2}\left(-0.71 \mathrm{~m} / \mathrm{s}^{2}\right)(3.2 \mathrm{~s})^{2} \\
\vec{v}_{0} & =-1.4 \mathrm{~m} / \mathrm{s} \text { or } 1.4 \mathrm{~m} / \mathrm{s} \text { left }
\end{aligned}
$$

23. 

| $\vec{v}_{\mathrm{i}}$ | $\vec{v}_{\mathrm{f}}$ | $\vec{a}$ | $\vec{d}$ | $t$ |
| :---: | :---: | :---: | :---: | :---: |
| $-15.0 \mathrm{~m} / \mathrm{s}$ | $-35.0 \mathrm{~m} / \mathrm{s}$ | $?$ | -43.0 m | $\times$ |

$$
\bar{v}_{\mathrm{f}}^{2}=\bar{v}_{\mathrm{i}}^{2}+2 \bar{a} \vec{d}
$$

$$
(-35.0 \mathrm{~m} / \mathrm{s})^{2}=(-15.0 \mathrm{~m} / \mathrm{s})^{2}+2 \vec{a}(-43.0 \mathrm{~m})
$$

$$
\vec{a}=-11.6 \mathrm{~m} / \mathrm{s}^{2} \text { or } 11.6 \mathrm{~m} / \mathrm{s}^{2} \mathrm{left}
$$

24. 

| $\vec{v}_{\mathrm{i}}$ | $\vec{v}_{\mathrm{f}}$ | $\vec{a}$ | $\vec{d}$ | $t$ |
| :---: | :---: | :---: | :---: | :---: |
| $-7.0 \mathrm{~m} / \mathrm{s}$ | $-19.0 \mathrm{~m} / \mathrm{s}$ | $\times$ | $\times$ | $\times$ |

$$
\begin{aligned}
\vec{v}_{\text {ave }} & =\frac{\vec{v}_{\mathrm{f}}+\vec{v}_{\mathrm{i}}}{2} \\
& =\frac{((-7.0 \mathrm{~m} / \mathrm{s})+(-19.0 \mathrm{~m} / \mathrm{s}))}{2} \\
& =-13.0 \mathrm{~m} / \mathrm{s} \text { or } 13.0 \mathrm{~m} / \mathrm{s} \text { left }
\end{aligned}
$$

25. 

| $\vec{v}_{\mathrm{i}}$ | $\vec{v}_{\mathrm{f}}$ | $\vec{a}$ | $\vec{d}$ | $t$ |
| :---: | :---: | :---: | :---: | :---: |
| $?$ | $25.0 \mathrm{~m} / \mathrm{s}$ | $1.50 \mathrm{~m} / \mathrm{s}^{2}$ | $\times$ | 10.0 s |

$$
\begin{aligned}
\vec{a} & =\frac{\vec{v}_{\mathrm{f}}-\vec{v}_{\mathrm{i}}}{t} \\
1.50 \mathrm{~m} / \mathrm{s}^{2} & =\frac{25.0 \mathrm{~m} / \mathrm{s}-\vec{v}_{\mathrm{i}}}{10.0 \mathrm{~s}} \\
\vec{v}_{\mathrm{i}} & =25.0 \mathrm{~m} / \mathrm{s}-(10.0 \mathrm{~s})\left(1.50 \mathrm{~m} / \mathrm{s}^{2}\right) \\
& =10.0 \mathrm{~m} / \mathrm{s} \text { right }
\end{aligned}
$$

26. 

| $\vec{v}_{\mathrm{i}}$ | $\vec{v}_{\mathrm{f}}$ | $\vec{a}$ | $\vec{d}$ | $t$ |
| :---: | :---: | :---: | :---: | :---: |
| $?$ | $14.0 \mathrm{~m} / \mathrm{s}$ | $\times$ | 25.0 m | 1.90 s |

$$
\vec{d}=\left(\frac{\stackrel{\rightharpoonup}{\mathrm{v}}_{\mathrm{f}}+\vec{v}_{\mathrm{i}}}{2}\right) t
$$

$25.0 \mathrm{~m}=\left(\frac{14.0 \mathrm{~m} / \mathrm{s}+\vec{v}_{\mathrm{i}}}{2}\right)(1.90 \mathrm{~s})$

$$
\begin{aligned}
\stackrel{\rightharpoonup}{v}_{\mathrm{i}} & =\frac{2(25.0 \mathrm{~m})}{1.90 \mathrm{~s}}-14.0 \mathrm{~m} / \mathrm{s} \\
& =12.3 \mathrm{~m} / \mathrm{s} \text { right }
\end{aligned}
$$

27. 

| $\vec{v}_{\mathrm{i}}$ | $\vec{v}_{\mathrm{f}}$ | $\vec{a}$ | $\vec{d}$ | $t$ |
| :---: | :---: | :---: | :---: | :---: |
| 0 | $?$ | $\times$ | -31.0 m | 5.6 s |

$$
\begin{aligned}
\vec{d} & =\left(\frac{\vec{v}_{\mathrm{f}}+\vec{v}_{\mathrm{i}}}{2}\right) t \\
-31.0 \mathrm{~m} & =\left(\frac{\vec{v}_{\mathrm{f}}+0}{2}\right) 5.6 \mathrm{~s} \\
\vec{v}_{\mathrm{f}} & =\frac{2(-31.0 \mathrm{~m})}{5.6 \mathrm{~s}} \\
& =-11 \mathrm{~m} / \mathrm{s} \text { or } 11 \mathrm{~m} / \mathrm{s} \text { left }
\end{aligned}
$$

28. 

| $\vec{v}_{\mathrm{i}}$ | $\vec{v}_{\mathrm{f}}$ | $\vec{a}$ | $\vec{d}$ | $t$ |
| :---: | :---: | :---: | :---: | :---: |
| $?$ | $-25.0 \mathrm{~m} / \mathrm{s}$ | $-0.900 \mathrm{~m} / \mathrm{s}^{2}$ | -37.0 m | $\times$ |

$$
\vec{v}_{\mathrm{f}}^{2}=\vec{v}_{\mathrm{i}}^{2}+2 \vec{a} \vec{d}
$$

$$
(-25.0 \mathrm{~m} / \mathrm{s})^{2}=\bar{v}_{\mathrm{i}}^{2}+2\left(-0.900 \mathrm{~m} / \mathrm{s}^{2}\right)(-37.0 \mathrm{~m})
$$

$$
\vec{v}_{\mathrm{i}}=-23.6 \mathrm{~m} / \mathrm{s} \text { or } 23.6 \mathrm{~m} / \mathrm{s} \text { left }
$$

29. 

| $\vec{v}_{\mathrm{i}}$ | $\vec{v}_{\mathrm{f}}$ | $\vec{a}$ | $\vec{d}$ | $t$ |
| :---: | :---: | :---: | :---: | :---: |
| 0 | $?$ | $1.4 \mathrm{~m} / \mathrm{s}^{2}$ | $?$ | 5.0 s |

$$
\text { a) } \quad \begin{aligned}
\vec{d} & =\vec{v}_{\mathrm{i}} t+\frac{1}{2} \vec{a} t^{2} \\
& =\frac{1}{2}\left(1.4 \mathrm{~m} / \mathrm{s}^{2}\right)(5.0 \mathrm{~s})^{2} \\
& =18 \mathrm{~m} \text { down the ramp }
\end{aligned}
$$

b) $\quad \vec{a}=\frac{\vec{v}_{\mathrm{f}}-\vec{v}_{\mathrm{i}}}{t}$
$1.4 \mathrm{~m} / \mathrm{s}^{2}=\frac{\vec{v}_{\mathrm{f}}-0}{5.0 \mathrm{~s}}$
$\vec{v}_{\mathrm{f}}=7.0 \mathrm{~m} / \mathrm{s}$ down the ramp
30. $\vec{v}_{\mathrm{ave}}=\frac{\vec{d}}{t}$

$$
\begin{aligned}
\stackrel{\rightharpoonup}{d} & =\stackrel{\rightharpoonup}{v}_{\text {ave }} t \\
& =(9.60 \mathrm{~m} / \mathrm{s})(2.70 \mathrm{~s}) \\
& =25.9 \mathrm{~m} \text { right }
\end{aligned}
$$

31. 

| $\vec{v}_{\mathrm{i}}$ | $\vec{v}_{\mathrm{f}}$ | $\vec{a}$ | $\vec{d}$ | $t$ |
| :---: | :---: | :---: | :---: | :---: |
| 0 | $-12.4 \mathrm{~m} / \mathrm{s}$ | $-3.10 \mathrm{~m} / \mathrm{s}^{2}$ | $?$ | 4.00 s |

You can use any formula except
$\vec{a}=\frac{\vec{v}_{\mathrm{f}}-\vec{v}_{\mathrm{i}}}{t}$
$\vec{d}=\vec{v}_{\mathrm{i}} t+\frac{1}{2} \vec{a} t^{2}$
$=\frac{1}{2}\left(-3.10 \mathrm{~m} / \mathrm{s}^{2}\right)(4.00 \mathrm{~s})^{2}$
$=-24.8 \mathrm{~m}$ or 24.8 m left
32.

| $\vec{v}_{\mathrm{i}}$ | $\vec{v}_{\mathrm{f}}$ | $\vec{a}$ | $\vec{d}$ | $t$ |
| :---: | :---: | :---: | :---: | :---: |
| 0 | $-10.0 \mathrm{~m} / \mathrm{s}$ | $?$ | -12.5 m | 2.50 s |

You can use any formula except $\vec{d}=\left(\frac{\vec{v}_{\mathrm{f}}+\vec{v}_{\mathrm{i}}}{2}\right) t$

$$
\begin{aligned}
\vec{a} & =\frac{\vec{v}_{\mathrm{f}}-\vec{v}_{\mathrm{i}}}{t} \\
& =\frac{(-10.0 \mathrm{~m} / \mathrm{s})-0}{2.50 \mathrm{~s}} \\
& =-4.00 \mathrm{~m} / \mathrm{s}^{2} \text { or } 4.00 \mathrm{~m} / \mathrm{s}^{2} \mathrm{left}
\end{aligned}
$$

33. 

| $\vec{v}_{\mathrm{i}}$ | $\vec{v}_{\mathrm{f}}$ | $\vec{a}$ | $\vec{d}$ | $t$ |
| :---: | :---: | :---: | :---: | :---: |
| 0 | $\times$ | $?$ | 19.6 m | $?$ |

$$
\begin{aligned}
\stackrel{\rightharpoonup}{v}_{\text {ave }} & =\frac{\stackrel{\rightharpoonup}{d}}{t} \\
t & =\frac{\vec{d}}{\vec{v}_{\text {ave }}} \\
& =\frac{19.6 \mathrm{~m}}{5.00 \mathrm{~m} / \mathrm{s}} \\
& =3.92 \mathrm{~s}
\end{aligned}
$$

$$
\vec{d}=\vec{v}_{\mathrm{i}} t+\frac{1}{2} \vec{a} t^{2}
$$

$$
19.6 \mathrm{~m}=\frac{1}{2} \vec{a}(3.92 \mathrm{~s})^{2}
$$

$$
\vec{a}=2.55 \mathrm{~m} / \mathrm{s}^{2} \text { right }
$$

34. 

| $\vec{v}_{\mathrm{i}}$ | $\vec{v}_{\mathrm{f}}$ | $\vec{a}$ | $\vec{d}$ | $t$ |
| :---: | :---: | :---: | :---: | :---: |
| $\times$ | $?$ | $1.00 \mathrm{~m} / \mathrm{s}^{2}$ | 417.2 m | 27.0 s |

Find initial velocity first:

$$
\begin{aligned}
\vec{d} & =\vec{v}_{\mathrm{i}} t+\frac{1}{2} \vec{a} t^{2} \\
417.2 \mathrm{~m} & =\vec{v}_{\mathrm{i}}(27.0 \mathrm{~s})+\frac{1}{2}\left(1.00 \mathrm{~m} / \mathrm{s}^{2}\right)(27.0 \mathrm{~s})^{2} \\
\vec{v}_{\mathrm{i}} & =\frac{417.2 \mathrm{~m}-\frac{1}{2}\left(1.00 \mathrm{~m} / \mathrm{s}^{2}\right)(27.0 \mathrm{~s})^{2}}{27.0 \mathrm{~s}} \\
& =1.952 \mathrm{~m} / \mathrm{s} \text { right } \\
\vec{a} & =\frac{\vec{v}_{\mathrm{f}}-\vec{v}_{\mathrm{i}}}{t} \\
1.00 \mathrm{~m} / \mathrm{s}^{2} & =\frac{\vec{v}_{\mathrm{f}}-1.952 \mathrm{~m} / \mathrm{s}}{27.0 \mathrm{~s}} \\
\vec{v}_{\mathrm{f}} & =(27.0 \mathrm{~s})\left(1.00 \mathrm{~m} / \mathrm{s}^{2}\right)+1.952 \mathrm{~m} / \mathrm{s} \\
& =29.0 \mathrm{~m} / \mathrm{s} \text { right }
\end{aligned}
$$

## Lesson 4-Freely Falling Objects

## PRACTICE EXERCISES

 ANSWERS AND SOLUTIONSNote: in all of the following questions, consider up as the positive direction and down as the negative direction.
1.

| $\vec{v}_{\mathrm{i}}$ | $\vec{v}_{\mathrm{f}}$ | $\vec{a}$ | $\vec{d}$ | $t$ |
| :---: | :---: | :---: | :---: | :---: |
| 0 | $?$ | $-9.81 \mathrm{~m} / \mathrm{s}^{2}$ | -15.0 m | $\times$ |

$\vec{v}_{\mathrm{f}}^{2}=\vec{v}_{\mathrm{i}}^{2}+2 \vec{a} \vec{d}$
$=2\left(-9.81 \mathrm{~m} / \mathrm{s}^{2}\right)(-15.0 \mathrm{~m})$
$\vec{v}_{\mathrm{f}}=-17.2 \mathrm{~m} / \mathrm{s}$ or $17.2 \mathrm{~m} / \mathrm{s}$ down
2.

| $\vec{v}_{\mathrm{i}}$ | $\vec{v}_{\mathrm{f}}$ | $\vec{a}$ | $\vec{d}$ | $t$ |
| :---: | :---: | :---: | :---: | :---: |
| 0 | $\times$ | $-9.81 \mathrm{~m} / \mathrm{s}^{2}$ | $?$ | 0.50 s |

$$
\begin{aligned}
\vec{d} & =\vec{v}_{\mathrm{i}} t+\frac{1}{2} \vec{a} t^{2} \\
& =\frac{1}{2}\left(-9.81 \mathrm{~m} / \mathrm{s}^{2}\right)(0.50 \mathrm{~s})^{2} \\
& =-1.2 \mathrm{~m} \text { or } 1.2 \mathrm{~m} \text { down }
\end{aligned}
$$

The apple falls from a height of 1.2 m .
3.

| $\vec{v}_{\mathrm{i}}$ | $\vec{v}_{\mathrm{f}}$ | $\vec{a}$ | $\vec{d}$ | $t$ |
| :---: | :---: | :---: | :---: | :---: |
| 0 | $\times$ | $-9.81 \mathrm{~m} / \mathrm{s}^{2}$ | -1.75 m | $?$ |

$$
\begin{aligned}
\vec{d} & =\vec{v}_{\mathrm{i}} t+\frac{1}{2} \vec{a} t^{2} \\
-1.75 \mathrm{~m} & =\frac{1}{2}\left(-9.81 \mathrm{~m} / \mathrm{s}^{2}\right) t^{2} \\
t & =\sqrt{\frac{2(-1.75 \mathrm{~m})}{-9.81 \mathrm{~m} / \mathrm{s}^{2}}} \\
t & =0.597 \mathrm{~s}
\end{aligned}
$$

4. 

| $\vec{v}_{\mathrm{i}}$ | $\vec{v}_{\mathrm{f}}$ | $\vec{a}$ | $\vec{d}$ | $t$ |
| :---: | :---: | :---: | :---: | :---: |
| 0 | $?$ | $-9.81 \mathrm{~m} / \mathrm{s}^{2}$ | -9.50 m | $\times$ |

$\vec{v}_{\mathrm{f}}^{2}=\vec{v}_{\mathrm{i}}^{2}+2 \vec{a} \vec{d}$
$=2\left(-9.81 \mathrm{~m} / \mathrm{s}^{2}\right)(-9.50 \mathrm{~m})$
$\vec{v}_{\mathrm{f}}=-13.7 \mathrm{~m} / \mathrm{s}$ or $13.7 \mathrm{~m} / \mathrm{s}$ down
5.

| $\vec{v}_{\mathrm{i}}$ | $\vec{v}_{\mathrm{f}}$ | $\vec{a}$ | $\vec{d}$ | $t$ |
| :---: | :---: | :---: | :---: | :---: |
| 0 | $?$ | $-9.81 \mathrm{~m} / \mathrm{s}^{2}$ | $\times$ | 2.5 s |

$$
\begin{aligned}
\vec{a} & =\frac{\stackrel{\rightharpoonup}{\mathrm{f}}_{\mathrm{f}}-\vec{v}_{\mathrm{i}}}{t} \\
-9.81 \mathrm{~m} / \mathrm{s}^{2} & =\frac{\stackrel{\rightharpoonup}{\mathrm{f}}_{\mathrm{f}}-0}{2.5 \mathrm{~s}} \\
\stackrel{\rightharpoonup}{v}_{\mathrm{f}} & =-25 \mathrm{~m} / \mathrm{s} \text { or } 25 \mathrm{~m} / \mathrm{s} \text { down }
\end{aligned}
$$

6. 

| $\vec{v}_{\mathrm{i}}$ | $\vec{v}_{\mathrm{f}}$ | $\vec{a}$ | $\vec{d}$ | $t$ |
| :---: | :---: | :---: | :---: | :---: |
| $?$ | $\times$ | $-9.81 \mathrm{~m} / \mathrm{s}^{2}$ | -11.2 m | 0.550 s |

$$
\begin{aligned}
\vec{d} & =\vec{v}_{\mathrm{i}} t+\frac{1}{2} \vec{a} t^{2} \\
-11.2 \mathrm{~m} & =\vec{v}_{\mathrm{i}}(0.550 \mathrm{~s})+\frac{1}{2}\left(-9.81 \mathrm{~m} / \mathrm{s}^{2}\right)(0.550 \mathrm{~s})^{2} \\
\vec{v}_{\mathrm{i}} & =\frac{(-11.2 \mathrm{~m})-\frac{1}{2}\left(-9.81 \mathrm{~m} / \mathrm{s}^{2}\right)(0.550 \mathrm{~s})^{2}}{0.550 \mathrm{~s}} \\
& =-17.7 \mathrm{~m} / \mathrm{s} \text { or } 17.7 \mathrm{~m} / \mathrm{s} \text { down }
\end{aligned}
$$

7. 

| $\vec{v}_{\mathrm{i}}$ | $\vec{v}_{\mathrm{f}}$ | $\vec{a}$ | $\vec{d}$ | $t$ |
| :---: | :---: | :---: | :---: | :---: |
| $-10.0 \mathrm{~m} / \mathrm{s}$ | $-25.0 \mathrm{~m} / \mathrm{s}$ | $-9.81 \mathrm{~m} / \mathrm{s}^{2}$ | $\times$ | $?$ |

$$
\begin{aligned}
\vec{a} & =\frac{\vec{v}_{\mathrm{f}}-\vec{v}_{\mathrm{i}}}{t} \\
-9.81 \mathrm{~m} / \mathrm{s}^{2} & =\frac{(-25.0 \mathrm{~m} / \mathrm{s})-(-10.0 \mathrm{~m} / \mathrm{s})}{t} \\
t & =\frac{(-25.0 \mathrm{~m} / \mathrm{s})-(-10.0 \mathrm{~m} / \mathrm{s})}{-9.81 \mathrm{~m} / \mathrm{s}^{2}} \\
& =1.53 \mathrm{~s}
\end{aligned}
$$

8. 

| $\vec{v}_{\mathrm{i}}$ | $\vec{v}_{\mathrm{f}}$ | $\vec{a}$ | $\vec{d}$ | $t$ |
| :---: | :---: | :---: | :---: | :---: |
| $-5.0 \mathrm{~m} / \mathrm{s}$ | $-15.0 \mathrm{~m} / \mathrm{s}$ | $-9.81 \mathrm{~m} / \mathrm{s}^{2}$ | $?$ | $\times$ |

$$
\vec{v}_{\mathrm{f}}^{2}=\vec{v}_{\mathrm{i}}^{2}+2 \vec{a} \vec{d}
$$

$$
(-15.0 \mathrm{~m} / \mathrm{s})^{2}=(-5.0 \mathrm{~m} / \mathrm{s})^{2}+2\left(-9.81 \mathrm{~m} / \mathrm{s}^{2}\right) \vec{d}
$$

$$
\vec{d}=\frac{(-15.0 \mathrm{~m} / \mathrm{s})^{2}-(-5.0 \mathrm{~m} / \mathrm{s})^{2}}{2\left(-9.81 \mathrm{~m} / \mathrm{s}^{2}\right)}
$$

$$
=-1.0 \times 10^{1} \mathrm{~m} \text { or } 1.0 \times 10^{1} \mathrm{~m} \text { down }
$$

Therefore, the rock was released $1.0 \times 10^{1} \mathrm{~m}$ above the ground.
9.

| $\vec{v}_{\mathrm{i}}$ | $\vec{v}_{\mathrm{f}}$ | $\vec{a}$ | $\vec{d}$ | $t$ |
| :---: | :---: | :---: | :---: | :---: |
| $?$ | $-10.0 \mathrm{~m} / \mathrm{s}$ | $-9.81 \mathrm{~m} / \mathrm{s}^{2}$ | $\times$ | 0.880 s |

$$
\begin{aligned}
\vec{a} & =\frac{\vec{v}_{\mathrm{f}}-\vec{v}_{\mathrm{i}}}{t} \\
-9.81 \mathrm{~m} / \mathrm{s}^{2} & =\frac{(-10.0 \mathrm{~m} / \mathrm{s})-\vec{v}_{\mathrm{i}}}{0.880 \mathrm{~s}} \\
\vec{v}_{\mathrm{i}} & =-1.37 \mathrm{~m} / \mathrm{s} \text { or } 1.37 \mathrm{~m} / \mathrm{s} \text { down }
\end{aligned}
$$

10. 

| $\vec{v}_{\mathrm{i}}$ | $\vec{v}_{\mathrm{f}}$ | $\vec{a}$ | $\vec{d}$ | $t$ |
| :---: | :---: | :---: | :---: | :---: |
| 0 | $\times$ | $-9.81 \mathrm{~m} / \mathrm{s}^{2}$ | -50.0 m | $?$ |

Find how far it falls in the 1st 2 seconds:

$$
\begin{aligned}
& \vec{d}=\vec{v}_{\mathrm{i}} t+\frac{1}{2} \vec{a} t^{2} \\
& =\frac{1}{2}\left(-9.81 \mathrm{~m} / \mathrm{s}^{2}\right)(2.00 \mathrm{~s})^{2} \\
& =-19.6 \mathrm{~m} \text { or } 19.6 \mathrm{~m} \text { down }
\end{aligned}
$$

Now, find how far it falls in the 1st 3 seconds:

$$
\begin{aligned}
\vec{d} & =\vec{v}_{\mathrm{i}} t+\frac{1}{2} \vec{a} t^{2} \\
& =\frac{1}{2}\left(-9.81 \mathrm{~m} / \mathrm{s}^{2}\right)(3.00 \mathrm{~s})^{2} \\
& =-44.1 \mathrm{~m} \text { or } 44.1 \mathrm{~m} \text { down }
\end{aligned}
$$

Difference is:

$$
\begin{aligned}
\vec{d} & =(-44.1 \mathrm{~m})-(-19.6 \mathrm{~m}) \\
& =-24.5 \mathrm{~m} \text { or } 24.5 \mathrm{~m} \text { down }
\end{aligned}
$$

11. a) $\vec{v}_{\text {ave }}=\frac{\vec{v}_{\mathrm{f}}+\vec{v}_{\mathrm{i}}}{2}$

$$
\begin{aligned}
& =\frac{50.0 \mathrm{~m} / \mathrm{s}+0}{2} \\
& =25.0 \mathrm{~m} / \mathrm{s} \text { left }
\end{aligned}
$$

b) Read time from the graph.
$t=4.0 \mathrm{~s}$
12.

| $\vec{v}_{\mathrm{i}}$ | $\vec{v}_{\mathrm{f}}$ | $\vec{a}$ | $\vec{d}$ | $t$ |
| :---: | :---: | :---: | :---: | :---: |
| 0 | $\times$ | $-9.81 \mathrm{~m} / \mathrm{s}^{2}$ | -7.0 m | $\times$ |

a) Find either final velocity:

$$
\begin{aligned}
\vec{v}_{\mathrm{f}}^{2} & =\vec{v}_{\mathrm{i}}^{2}+2 \vec{a} \vec{d} \\
& =2\left(-9.81 \mathrm{~m} / \mathrm{s}^{2}\right)(-7.0 \mathrm{~m}) \\
\vec{v}_{\mathrm{f}} & =-11.7 \mathrm{~m} / \mathrm{s} \text { or } 11.7 \mathrm{~m} / \mathrm{s} \text { down } \\
\vec{v}_{\mathrm{av}} & =\frac{\vec{v}_{\mathrm{f}}+\vec{v}_{\mathrm{i}}}{2} \\
& =\frac{(-11.7 \mathrm{~m} / \mathrm{s})+0}{2} \\
& =-5.9 \mathrm{~m} / \mathrm{s} \text { or } 5.9 \mathrm{~m} / \mathrm{s} \text { down }
\end{aligned}
$$

b) $\quad \vec{v}_{\mathrm{ave}}=\frac{\vec{d}}{t}$

$$
\begin{aligned}
t & =\frac{\stackrel{\rightharpoonup}{d}}{\vec{v}_{\text {ave }}} \\
& =\frac{-7.0 \mathrm{~m}}{-5.9 \mathrm{~m} / \mathrm{s}} \\
& =1.2 \mathrm{~s}
\end{aligned}
$$

c) The average velocity occurs at the mid-time in uniform accelerated motion. Therefore, the average velocity occurs at 0.60 s .
13. $\vec{v}_{\text {ave }}=\frac{\vec{v}_{\mathrm{f}}+\vec{v}_{\mathrm{i}}}{2}$
$-12.0 \mathrm{~m} / \mathrm{s}=\frac{\stackrel{\rightharpoonup}{\mathrm{v}}_{\mathrm{f}}-0}{2}$
$\vec{v}_{\mathrm{f}} \equiv-24.0 \mathrm{~m} / \mathrm{s}$ or $24.0 \mathrm{~m} / \mathrm{s}$ down

| $\vec{v}_{\mathrm{i}}$ | $\vec{v}_{\mathrm{f}}$ | $\vec{a}$ | $\vec{d}$ | $t$ |
| :---: | :---: | :---: | :---: | :---: |
| 0 | $-24.0 \mathrm{~m} / \mathrm{s}$ | $-9.81 \mathrm{~m} / \mathrm{s}^{2}$ | $\times$ | $?$ |

$$
\begin{aligned}
\vec{a} & =\frac{\vec{v}_{\mathrm{f}}-\vec{v}_{\mathrm{i}}}{t} \\
t & =\frac{(-24.0 \mathrm{~m} / \mathrm{s})-0}{-9.81 \mathrm{~m} / \mathrm{s}^{2}} \\
& =2.45 \mathrm{~s}
\end{aligned}
$$

14. 

| $\vec{v}_{\mathrm{i}}$ | $\vec{v}_{\mathrm{f}}$ | $\vec{a}$ | $\vec{d}$ | $t$ |
| :---: | :---: | :---: | :---: | :---: |
| 0 | $\times$ | $?$ | -10.0 m | 1.20 s |

$$
\begin{aligned}
\vec{d} & =\vec{v}_{\mathrm{i}} t+\frac{1}{2} \vec{a} t^{2} \\
-10.0 \mathrm{~m} & =\frac{1}{2} \vec{a}(1.20 \mathrm{~s})^{2} \\
\vec{a} & =\frac{2(-10.0 \mathrm{~m})}{(1.20 \mathrm{~s})^{2}} \\
& =-13.9 \mathrm{~m} / \mathrm{s}^{2} \text { or } 13.9 \mathrm{~m} / \mathrm{s}^{2} \text { down }
\end{aligned}
$$

15. 

| $\vec{v}_{\mathrm{i}}$ | $\vec{v}_{\mathrm{f}}$ | $\vec{a}$ | $\vec{d}$ | $t$ |
| :---: | :---: | :---: | :---: | :---: |
| 0 | $-19.6 \mathrm{~m} / \mathrm{s}$ | $?$ | -24.0 m | $\times$ |

$$
\begin{aligned}
\vec{v}_{\mathrm{f}}^{2} & =\vec{v}_{\mathrm{i}}^{2}+2 \vec{a} \vec{d} \\
(-19.6 \mathrm{~m} / \mathrm{s})^{2} & =2 \vec{a}(-24.0 \mathrm{~m}) \\
\vec{a} & =\frac{(-19.6 \mathrm{~m} / \mathrm{s})^{2}}{2(-24.0 \mathrm{~m})} \\
& =-8.00 \mathrm{~m} / \mathrm{s}^{2} \text { or } 8.00 \mathrm{~m} / \mathrm{s}^{2} \text { down }
\end{aligned}
$$

16. 

| $\vec{v}_{\mathrm{i}}$ | $\vec{v}_{\mathrm{f}}$ | $\vec{a}$ | $\vec{d}$ | $t$ |
| :---: | :---: | :---: | :---: | :---: |
| 0 | $-11.0 \mathrm{~m} / \mathrm{s}$ | $?$ | $\times$ | 1.50 s |

$\vec{a}=\frac{\stackrel{\rightharpoonup}{v}_{\mathrm{f}}-\vec{v}_{\mathrm{i}}}{t}$
$=\frac{(-11.0 \mathrm{~m} / \mathrm{s})-0}{1.50 \mathrm{~s}}$
$=-7.33 \mathrm{~m} / \mathrm{s}^{2}$ or $7.33 \mathrm{~m} / \mathrm{s}^{2}$ down
17.

| $\vec{v}_{\mathrm{i}}$ | $\vec{v}_{\mathrm{f}}$ | $\vec{a}$ | $\vec{d}$ | $t$ |
| :---: | :---: | :---: | :---: | :---: |
| $\times$ | $-15.0 \mathrm{~m} / \mathrm{s}$ | $-9.81 \mathrm{~m} / \mathrm{s}^{2}$ | -10.0 m | $?$ |

Find initial velocity first:

$$
\begin{aligned}
\vec{v}_{\mathrm{f}}^{2} & =\bar{v}_{\mathrm{i}}^{2}+2 \vec{a} \vec{d} \\
(-15.0 \mathrm{~m} / \mathrm{s})^{2} & =\vec{v}_{\mathrm{i}}^{2}+2\left(-9.81 \mathrm{~m} / \mathrm{s}^{2}\right)(-10.0 \mathrm{~m}) \\
\vec{v}_{\mathrm{i}} & =-5.367 \mathrm{~m} / \mathrm{s} \text { or } 5.367 \mathrm{~m} / \mathrm{s} \text { down }
\end{aligned}
$$

Now use any equation that you like to find time:

$$
\begin{aligned}
\vec{a} & =\frac{\vec{v}_{\mathrm{f}}-\vec{v}_{\mathrm{i}}}{t} \\
-9.81 \mathrm{~m} / \mathrm{s}^{2} & =\frac{(-15.0 \mathrm{~m} / \mathrm{s})-(-5.367 \mathrm{~m} / \mathrm{s})}{t} \\
t & =0.982 \mathrm{~s}
\end{aligned}
$$

18. 

| $\vec{v}_{\mathrm{i}}$ | $\vec{v}_{\mathrm{f}}$ | $\vec{a}$ | $\vec{d}$ | $t$ |
| :---: | :---: | :---: | :---: | :---: |
| $\times$ | $?$ | $-9.81 \mathrm{~m} / \mathrm{s}^{2}$ | -9.2 m | 0.85 s |

Find the initial velocity first:

$$
\begin{aligned}
\vec{d} & =\vec{v}_{\mathrm{i}} t+\frac{1}{2} \vec{a} t^{2} \\
-9.2 \mathrm{~m} & =\vec{v}_{\mathrm{i}}(0.85 \mathrm{~s})+\frac{1}{2}\left(-9.81 \mathrm{~m} / \mathrm{s}^{2}\right)(0.85 \mathrm{~s})^{2} \\
\vec{v}_{\mathrm{i}} & =\frac{(-9.2 \mathrm{~m})-\frac{1}{2}\left(-9.81 \mathrm{~m} / \mathrm{s}^{2}\right)(0.85 \mathrm{~s})^{2}}{0.85 \mathrm{~s}} \\
& =-6.65 \mathrm{~m} / \mathrm{s} \text { or } 6.65 \mathrm{~m} / \mathrm{s} \text { down }
\end{aligned}
$$

Now find final velocity:

$$
\begin{aligned}
\vec{a} & =\frac{\stackrel{\rightharpoonup}{\mathrm{f}}-\vec{v}_{\mathrm{i}}}{t} \\
\left(-9.81 \mathrm{~m} / \mathrm{s}^{2}\right) & =\frac{\stackrel{\rightharpoonup}{\mathrm{f}}-(-6.65 \mathrm{~m} / \mathrm{s})}{0.85 \mathrm{~s}} \\
\vec{v}_{\mathrm{f}} & =-15 \mathrm{~m} / \mathrm{s} \text { or } 15 \mathrm{~m} / \mathrm{s} \text { down }
\end{aligned}
$$

19. 

| $\vec{v}_{\mathrm{i}}$ | $\vec{v}_{\mathrm{f}}$ | $\vec{a}$ | $\vec{d}$ | $t$ |
| :---: | :---: | :---: | :---: | :---: |
| 0 | $?$ | $?$ | 40.0 m | $\times$ |

$$
\begin{aligned}
\vec{v}_{\mathrm{ave}} & =\frac{\vec{v}_{\mathrm{i}}+\vec{v}_{\mathrm{f}}}{2} \\
-30.0 \mathrm{~m} / \mathrm{s} & =\frac{0+\vec{v}_{\mathrm{f}}}{2} \\
\vec{v}_{\mathrm{f}} & =-60.0 \mathrm{~m} / \mathrm{s} \text { or } 60.0 \mathrm{~m} / \mathrm{s} \text { down } \\
\vec{v}_{\mathrm{f}}^{2} & =\vec{v}_{\mathrm{i}}^{2}+2 \vec{a} \vec{d} \\
(-60.0 \mathrm{~m} / \mathrm{s})^{2} & =2 \vec{a}(-40.0 \mathrm{~m}) \\
\vec{a} & =-45.0 \mathrm{~m} / \mathrm{s} \text { or } 45.0 \mathrm{~m} / \mathrm{s} \text { down }
\end{aligned}
$$

20. Average velocity during the 2 nd second is:

$$
\begin{aligned}
\stackrel{\rightharpoonup}{v}_{\text {ave }} & =\frac{\vec{d}}{t} \\
& =\frac{-5.00 \mathrm{~m}}{1.00 \mathrm{~s}} \\
& =-5.00 \mathrm{~m} / \mathrm{s} \text { or } 5.00 \mathrm{~m} / \mathrm{s} \text { down }
\end{aligned}
$$

This is the instantaneous velocity at $1.00 \mathrm{~s}+0.50 \mathrm{~s}=1.50 \mathrm{~s}$.

| $\vec{v}_{\mathrm{i}}$ | $\vec{v}_{\mathrm{f}}$ | $\vec{a}$ | $\vec{d}$ | $t$ |
| :---: | :---: | :---: | :---: | :---: |
| 0 | $-5.0 \mathrm{~m} / \mathrm{s}$ | $?$ | $\times$ | 1.50 s |

$$
\begin{aligned}
\vec{a} & =\frac{\vec{v}_{\mathrm{f}}-\vec{v}_{\mathrm{i}}}{t} \\
& =\frac{(-5.00 \mathrm{~m} / \mathrm{s})-0}{1.50 \mathrm{~s}} \\
& =-3.33 \mathrm{~m} / \mathrm{s}^{2} \text { or } 3.33 \mathrm{~m} / \mathrm{s}^{2} \text { down }
\end{aligned}
$$

## Lesson 5-Horizontal Uniformly Accelerated Motion in Two Directions along a Straight Line

## PRACTICE EXERCISES ANSWERS AND SOLUTIONS

1. Velocity is the slope of position-time graph. To find velocity, draw a tangent line at the point you are investigating and find the slope of the tangent line.
a) slope $=\frac{\text { rise }}{\text { run }}=\frac{y_{2}-y_{1}}{x_{2}-x_{1}}$

$$
\begin{aligned}
& =\frac{(14-5.0) \mathrm{m}}{(6.2-0) \mathrm{s}} \\
\stackrel{\rightharpoonup}{v} & =1.5 \mathrm{~m} / \mathrm{s} \mathrm{up}
\end{aligned}
$$

b) $\quad$ slope $=\frac{(5.0-14) \mathrm{m}}{(10.0-3.8) \mathrm{s}}$

$$
\vec{v}=-1.5 \mathrm{~m} / \mathrm{s} \text { or } 1.5 \mathrm{~m} / \mathrm{s} \text { down }
$$

2. Acceleration is the slope of the velocity-time graph. The slope is constant. Therefore, acceleration at 4.0 s and 10.0 s are equal.

$$
\begin{aligned}
\text { slope } & =\frac{\text { rise }}{\text { run }}=\frac{y_{2}-y_{1}}{x_{2}-x_{1}} \\
& =\frac{((-9.0)-9.0) \mathrm{m} / \mathrm{s}}{(12.0-2.0) \mathrm{s}} \\
\vec{a} & =-1.8 \mathrm{~m} / \mathrm{s}^{2} \text { or } 1.8 \mathrm{~m} / \mathrm{s}^{2} \text { left }
\end{aligned}
$$

3. a) Displacement is read from graph.
$\vec{d}=13.0 \mathrm{~m}$ right
b) Velocity is the slope of the position-time graph.

$$
\begin{aligned}
\text { slope } & =\frac{\text { rise }}{\text { run }}=\frac{y_{2}-y_{1}}{x_{2}-x_{1}} \\
& =\frac{(7.0-0) \mathrm{m}}{(7.0-0) \mathrm{s}} \\
\stackrel{\rightharpoonup}{v} & =1.0 \mathrm{~m} / \mathrm{s} \text { right }
\end{aligned}
$$

c) Velocity is the slope of the position-time graph.
slope $=0$
$\vec{v}=0$
d) Velocity is the slope of the position-time graph.

$$
\begin{aligned}
\text { slope } & =\frac{\text { rise }}{\text { run }}=\frac{y_{2}-y_{1}}{x_{2}-x_{1}} \\
& =\frac{(13.0-7.0) \mathrm{m}}{(16.0-13.0) \mathrm{s}} \\
\vec{v} & =2.0 \mathrm{~m} / \mathrm{s} \text { right }
\end{aligned}
$$

e) $\quad \vec{v}_{\mathrm{av}}=\frac{\vec{d}}{t}$

$$
\begin{aligned}
& =\frac{13.0 \mathrm{~m}}{16.0 \mathrm{~s}} \\
& =0.81 \mathrm{~m} / \mathrm{s} \text { right }
\end{aligned}
$$

f) Since slope does not change, the velocity does not change. $\vec{a}=0$
4. a) Read the velocity from the velocity-time graph.
$\vec{v}=4.0 \mathrm{~m} / \mathrm{s}$ right
b Read the velocity from a velocity-time graph. $\vec{v}=6.0 \mathrm{~m} / \mathrm{s}$ right
c) Read the velocity from a velocity-time graph. $\vec{v}=-3.0 \mathrm{~m} / \mathrm{s}$ or $3.0 \mathrm{~m} / \mathrm{s}$ left
d) Acceleration is the slope of the velocity-time graph.

$$
\begin{aligned}
\text { slope } & =\frac{\text { rise }}{\text { run }}=\frac{y_{2}-y_{1}}{x_{2}-x_{1}} \\
& =\frac{(-6.0-6.0) \mathrm{m} / \mathrm{s}}{(12.0-8.0) \mathrm{s}} \\
\vec{a} & =-3.0 \mathrm{~m} / \mathrm{s}^{2} \text { or } 3.0 \mathrm{~m} / \mathrm{s}^{2} \text { left }
\end{aligned}
$$

e) Acceleration is the slope of the velocity-time graph.

$$
\begin{aligned}
\text { slope } & =\frac{\text { rise }}{\text { run }}=\frac{y_{2}-y_{1}}{x_{2}-x_{1}} \\
& =\frac{(0-(-6.0)) \mathrm{m} / \mathrm{s}}{(20.0-14.0) \mathrm{s}} \\
\vec{a} & =1.0 \mathrm{~m} / \mathrm{s}^{2} \text { right }
\end{aligned}
$$

f) Displacement is the area under the velocitytime graph.

$$
\begin{aligned}
\text { area } & =\frac{1}{2}(l \times w)+(l \times w)+\frac{1}{2}(l \times w) \\
& =\frac{1}{2}(6.0 \times 6.0)+(6.0 \times 2.0)+\frac{1}{2}(6.0 \times 2.0) \\
\vec{d} & =36.0 \mathrm{~m} \text { right }
\end{aligned}
$$

g) $\quad \vec{v}_{\mathrm{av}}=\frac{\vec{d}}{t}$

Find the total displacement.
The displacement during the first 10.0 s is 36 m right.

Find the displacement during the last 10.0 s .

$$
\begin{aligned}
\text { area } & =\frac{1}{2}(l \times w)+(l \times w)+\frac{1}{2}(l \times w) \\
& =\binom{\frac{1}{2}(-6.0 \times 6.0)+(-6.0 \times 2.0)}{+\frac{1}{2}(-6.0 \times 2.0)} \\
\vec{d} & =-36.0 \mathrm{~m} \text { or } 36.0 \mathrm{~m} \mathrm{left}
\end{aligned}
$$

$$
\begin{aligned}
\vec{d} & =36.0-36.0 \\
& =0 \\
& \therefore \vec{v}=0
\end{aligned}
$$

5. a) Velocity is the slope of the position-time graph.

$$
\begin{aligned}
\text { slope } & =\frac{\text { rise }}{\text { run }}=\frac{y_{2}-y_{1}}{x_{2}-x_{1}} \\
& =\frac{(-16.0-16.0) \mathrm{m}}{(8.0-0) \mathrm{s}} \\
\vec{v} & =-4.0 \mathrm{~m} / \mathrm{s} \text { or } 4.0 \mathrm{~m} / \mathrm{s} \text { left }
\end{aligned}
$$

b) the slope is the same as part a).
$\vec{v}=-4.0 \mathrm{~m} / \mathrm{s}$ or $4.0 \mathrm{~m} / \mathrm{s}$ left
c) Velocity does not change.
$\vec{a}=0$
6. a) Read the velocity from a velocity-time graph.
$\vec{v}=14.6 \mathrm{~m} / \mathrm{s}$ right
b) Acceleration is the slope of the velocity-time graph.
slope $=\frac{\text { rise }}{\text { run }}=\frac{y_{2}-y_{1}}{x_{2}-x_{1}}$

$$
\vec{a}=0
$$

c) Displacement is the area under the velocitytime graph.
area $=l \times w$

$$
\begin{aligned}
& =10.0 \mathrm{~s} \times 15.0 \mathrm{~m} / \mathrm{s} \\
\bar{d} & =1.50 \times 10^{2} \mathrm{~m} \mathrm{right}
\end{aligned}
$$

7. a) Displacement is the area under the velocitytime graph.
greatest area $=\mathrm{A}$
b) greatest speed $=B$
8. a) Velocity is the slope of the position-time graph.
slope $=\frac{\text { rise }}{\text { run }}=\frac{y_{2}-y_{1}}{x_{2}-x_{1}}$
$=\frac{(40-(-40)) \mathrm{m}}{(11.0-5.0) \mathrm{s}}$
$\vec{v}=13 \mathrm{~m} / \mathrm{s}$ right
b) Velocity is the slope of the position-time graph. slope $=0$
$\stackrel{\rightharpoonup}{v}=0$
c) Read the displacement from the graph.
$\vec{d}=80.0 \mathrm{~m}$ right
9. a) D
b) A
c) C
d) $\mathrm{B}, \mathrm{D}$
10. a) i) Velocity is the slope of the position-time graph.

$$
\begin{aligned}
\text { slope } & =\frac{\text { rise }}{\text { run }}=\frac{y_{2}-y_{1}}{x_{2}-x_{1}} \\
& =\frac{(10.0-95.0) \mathrm{m}}{(20.0-15.0) \mathrm{s}} \\
\stackrel{\rightharpoonup}{v} & =-17.0 \mathrm{~m} / \mathrm{s} \text { or } 17.0 \mathrm{~m} / \mathrm{s} \text { left }
\end{aligned}
$$

ii) $d=A+B+C+D+E$

$$
\begin{aligned}
& =40 \mathrm{~m}+25 \mathrm{~m}+0+85 \mathrm{~m}+20 \mathrm{~m} \\
& =1.70 \times 10^{2} \mathrm{~m}
\end{aligned}
$$

iii) Since the position at the end of motion is the same as the position at the beginning, the displacement is zero.

$$
\vec{d}=0
$$

iv)

$$
\begin{aligned}
v_{\mathrm{av}} & =\frac{d}{t} \\
& =\frac{1.70 \times 10^{2} \mathrm{~m}}{25.0 \mathrm{~s}} \\
& =6.80 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

b) i) Velocity is the slope of the position-time graph. negative slope $=\mathrm{D}$
ii) Velocity (speed) is the slope of the position-time graph.
steepest slope $=\mathrm{D}$
11. a) Read the velocity from the velocity-time graph.
greatest velocity $=\mathrm{A}$
b) Acceleration is the slope of the velocity-time graph.
slope $=$ zero during A
c) Acceleration is the slope of the velocity-time graph.
steepest slope $=\mathrm{D}$
12. a)

| Time |
| :---: | :---: | :---: | :---: |
| $(\mathbf{s})$ | \left\lvert\, | Displacement |
| :---: |
| from $\boldsymbol{t}=\mathbf{0}(\mathbf{m})$ | | Displacement |
| :---: |
| 0 |\right.

b) Position-Time Graph

c) Velocity-Time Graph


Time (s)
d) i) Velocity is the slope of the position-time graph.

Draw a tangent line at 0.40 s and 1.10 s .

- 0.40 s

$$
\begin{aligned}
\text { slope } & =\frac{\text { rise }}{\text { run }}=\frac{y_{2}-y_{1}}{x_{2}-x_{1}} \\
& =\frac{(0.120-0.032) \mathrm{m}}{(0.56-0) \mathrm{s}} \\
\bar{v} & =0.16 \mathrm{~m} / \mathrm{s} \mathrm{up}
\end{aligned}
$$

- 1.10 s

$$
\begin{aligned}
\text { slope } & =\frac{\text { rise }}{\text { run }}=\frac{y_{2}-y_{1}}{x_{2}-x_{1}} \\
& =\frac{(0.020-0.120) \mathrm{m}}{(1.60-0.95) \mathrm{s}} \\
& =-0.15 \mathrm{~m} / \mathrm{s} \text { or } 0.15 \mathrm{~m} / \mathrm{s} \text { down }
\end{aligned}
$$

ii) Displacement is the area under the velocity-time graph.

$$
\begin{aligned}
\text { area } & =\frac{1}{2}(l \times w)+\frac{1}{2}(l \times w) \\
& =\binom{\frac{1}{2}(0.71 \mathrm{~s} \times 0.30 \mathrm{~m} / \mathrm{s})}{+\frac{1}{2}(0.64 \mathrm{~s} \times(-0.26 \mathrm{~m} / \mathrm{s}))} \\
& =0.107 \mathrm{~m}+(-0.083 \mathrm{~m}) \\
& =0.024 \mathrm{~m} \mathrm{up}
\end{aligned}
$$

iii) Acceleration is the slope of the velocity-time graph.

$$
\begin{aligned}
\text { slope } & =\frac{\text { rise }}{\text { run }}=\frac{y_{2}-y_{1}}{x_{2}-x_{1}} \\
& =\frac{(-0.25-0.30) \mathrm{m} / \mathrm{s}}{(1.35-0.050) \mathrm{s}} \\
\vec{a} & =-0.42 \mathrm{~m} / \mathrm{s}^{2} \text { or } 0.42 \mathrm{~m} / \mathrm{s}^{2} \text { down }
\end{aligned}
$$

## Lesson 6-Vertical Uniformly Accelerated Motion in Two

 Directions along a Straight LinePRACTICE EXERCISES ANSWERS AND SOLUTIONS
1.

| $\vec{v}_{0}$ | $\vec{v}_{\mathrm{f}}$ | $\vec{a}$ | $\vec{d}$ | $t$ |
| :---: | :---: | :---: | :---: | :---: |
| $14.0 \mathrm{~m} / \mathrm{s}$ | $\times$ | $-9.81 \mathrm{~m} / \mathrm{s}^{2}$ | $?$ | 1.80 s |

$$
\begin{aligned}
\vec{d} & =\vec{v}_{0} t+\frac{1}{2} \vec{a} t^{2} \\
& =(14.0 \mathrm{~m} / \mathrm{s})(1.80 \mathrm{~s})+\frac{1}{2}\left(-9.81 \mathrm{~m} / \mathrm{s}^{2}\right)(1.80 \mathrm{~s})^{2} \\
& =9.31 \mathrm{~m} \text { upward }
\end{aligned}
$$

2. 

| $\vec{v}_{0}$ | $\vec{v}_{\mathrm{f}}$ | $\vec{a}$ | $\vec{d}$ | $t$ |
| :---: | :---: | :---: | :---: | :---: |
| $9.3 \mathrm{~m} / \mathrm{s}$ | $\times$ | $?$ | 1.9 m | 2.7 s |

$$
\begin{aligned}
\vec{d} & =\vec{v}_{0} t+\frac{1}{2} \vec{a} t^{2} \\
1.9 \mathrm{~m} & =(9.3 \mathrm{~m} / \mathrm{s})(2.7 \mathrm{~s})+\frac{1}{2} \vec{a}(2.7 \mathrm{~s})^{2} \\
\vec{a} & =\frac{1.9 \mathrm{~m}-(9.3 \mathrm{~m} / \mathrm{s})(2.7 \mathrm{~s})}{\frac{1}{2}(2.7 \mathrm{~s})^{2}} \\
& =-6.4 \mathrm{~m} / \mathrm{s}^{2} \text { or } 6.4 \mathrm{~m} / \mathrm{s}^{2} \text { down the slope }
\end{aligned}
$$

3. 

| $\vec{v}_{0}$ | $\vec{v}_{\mathrm{f}}$ | $\vec{a}$ | $\vec{d}$ | $t$ |
| :---: | :---: | :---: | :---: | :---: |
| $11.0 \mathrm{~m} / \mathrm{s}$ | $-7.3 \mathrm{~m} / \mathrm{s}$ | $?$ | $\times$ | 9.3 s |

$$
\vec{a}=\frac{\stackrel{\rightharpoonup}{\mathrm{v}}_{\mathrm{f}}-\vec{v}_{0}}{t}
$$

$$
=\frac{(-7.3 \mathrm{~m} / \mathrm{s})-11.0 \mathrm{~m} / \mathrm{s}}{9.3 \mathrm{~s}}
$$

$$
=-2.0 \mathrm{~m} / \mathrm{s}^{2} \text { or } 2.0 \mathrm{~m} / \mathrm{s}^{2} \text { down the slope }
$$

4. 

| $\vec{v}_{0}$ | $\vec{v}_{\mathrm{f}}$ | $\vec{a}$ | $\vec{d}$ | $t$ |
| :---: | :---: | :---: | :---: | :---: |
| $9.4 \mathrm{~m} / \mathrm{s}$ | $-7.4 \mathrm{~m} / \mathrm{s}$ | $\times$ | $?$ | 3.0 s |

$$
\begin{aligned}
\vec{d} & =\left(\frac{\stackrel{\rightharpoonup}{\mathrm{f}}_{\mathrm{f}}+\vec{v}_{0}}{2}\right) t \\
& =\left(\frac{(-7.4 \mathrm{~m} / \mathrm{s})+9.4 \mathrm{~m} / \mathrm{s}}{2}\right)(3.0 \mathrm{~s}) \\
& =3.0 \mathrm{~m} \text { up the slope }
\end{aligned}
$$

5. 

| $\vec{v}_{0}$ | $\vec{v}_{\mathrm{f}}$ | $\vec{a}$ | $\vec{d}$ | $t$ |
| :---: | :---: | :---: | :---: | :---: |
| $15.0 \mathrm{~m} / \mathrm{s}$ | $-8.0 \mathrm{~m} / \mathrm{s}$ | $-9.81 \mathrm{~m} / \mathrm{s}^{2}$ | $?$ | $\times$ |

$$
\bar{v}_{\mathrm{f}}^{2}=\bar{v}_{0}^{2}+2 \bar{a} \vec{d}
$$

$$
(-8.0 \mathrm{~m} / \mathrm{s})^{2}=(15.0 \mathrm{~m} / \mathrm{s})^{2}+2\left(-9.81 \mathrm{~m} / \mathrm{s}^{2}\right) \vec{d}
$$

$$
\vec{d}=\frac{(-8.0 \mathrm{~m} / \mathrm{s})^{2}-(15.0 \mathrm{~m} / \mathrm{s})^{2}}{2\left(-9.81 \mathrm{~m} / \mathrm{s}^{2}\right)}
$$

$$
=8.2 \mathrm{mup}
$$

6. 

| $\vec{v}_{0}$ | $\vec{v}_{\mathrm{f}}$ | $\vec{a}$ | $\vec{d}$ | $t$ |
| :---: | :---: | :---: | :---: | :---: |
| $-5.0 \mathrm{~m} / \mathrm{s}$ | $-12.0 \mathrm{~m} / \mathrm{s}$ | $-9.80 \mathrm{~m} / \mathrm{s}^{2}$ | $\times$ | $?$ |

$$
\begin{aligned}
\vec{a} & =\frac{\vec{v}_{\mathrm{f}}-\vec{v}_{0}}{t} \\
-9.81 \mathrm{~m} / \mathrm{s}^{2} & =\frac{(-12.0 \mathrm{~m} / \mathrm{s})-(-5.0 \mathrm{~m} / \mathrm{s})}{t}
\end{aligned}
$$

$$
t=0.71 \mathrm{~s}
$$

7. 

| $\vec{v}_{0}$ | $\vec{v}_{\mathrm{f}}$ | $\vec{a}$ | $\vec{d}$ | $t$ |
| :---: | :---: | :---: | :---: | :---: |
| $?$ | 0 | $\times$ | 2.6 m | 3.6 s |

$$
\begin{aligned}
\vec{d} & =\left(\frac{\stackrel{\rightharpoonup}{\mathrm{f}}+\vec{v}_{0}}{2}\right) t \\
2.6 \mathrm{~m} & =\left(\frac{0+\vec{v}_{0}}{2}\right)(3.6 \mathrm{~s}) \\
\vec{v}_{0} & =\frac{2(2.6 \mathrm{~m})}{3.6 \mathrm{~s}} \\
& =1.4 \mathrm{~m} / \mathrm{s} \text { up }
\end{aligned}
$$

8. 

| $\vec{v}_{0}$ | $\vec{v}_{\mathrm{f}}$ | $\vec{a}$ | $\vec{d}$ | $t$ |
| :---: | :---: | :---: | :---: | :---: |
| $10.0 \mathrm{~m} / \mathrm{s}$ | $?$ | $-9.81 \mathrm{~m} / \mathrm{s}^{2}$ | 5.0 m | $\times$ |

$\vec{v}_{\mathrm{f}}^{2}=\vec{v}_{0}^{2}+2 \vec{a} \vec{d}$
$=(10.0 \mathrm{~m} / \mathrm{s})^{2}+2\left(-9.81 \mathrm{~m} / \mathrm{s}^{2}\right)(5.0 \mathrm{~m})$
$\vec{v}_{\mathrm{f}}=-1.4 \mathrm{~m} / \mathrm{s}$ or $1.4 \mathrm{~m} / \mathrm{s}$ down
9.

| $\vec{v}_{0}$ | $\vec{v}_{\mathrm{f}}$ | $\vec{a}$ | $\vec{d}$ | $t$ |
| :---: | :---: | :---: | :---: | :---: |
| $25.0 \mathrm{~m} / \mathrm{s}$ | $?$ | $-9.81 \mathrm{~m} / \mathrm{s}^{2}$ | $\times$ | 3.0 s |

$$
\begin{aligned}
\vec{a} & =\frac{\vec{v}_{\mathrm{f}}-\vec{v}_{0}}{t} \\
-9.81 \mathrm{~m} / \mathrm{s}^{2} & =\frac{\vec{v}_{\mathrm{f}}-25.0 \mathrm{~m} / \mathrm{s}}{3.0 \mathrm{~s}} \\
\vec{v}_{\mathrm{f}} & =(3.0 \mathrm{~s})\left(-9.81 \mathrm{~m} / \mathrm{s}^{2}\right)+25.0 \mathrm{~m} / \mathrm{s} \\
\vec{v}_{\mathrm{f}} & =-4.4 \mathrm{~m} / \mathrm{s} \text { or } 4.4 \mathrm{~m} / \mathrm{s} \text { down }
\end{aligned}
$$

10. 

| $\vec{v}_{0}$ | $\vec{v}_{\mathrm{f}}$ | $\vec{a}$ | $\vec{d}$ | $t$ |
| :---: | :---: | :---: | :---: | :---: |
| $2.0 \mathrm{~m} / \mathrm{s}$ | $?$ | $\times$ | 2.8 m | 1.5 s |

$$
\begin{aligned}
\vec{d} & =\left(\frac{\stackrel{\rightharpoonup}{\mathrm{f}}_{\mathrm{f}}+\vec{v}_{0}}{2}\right) t \\
2.8 \mathrm{~m} & =\left(\frac{\vec{v}_{\mathrm{f}}+2.0 \mathrm{~m} / \mathrm{s}}{2}\right)(1.5 \mathrm{~s}) \\
\vec{v}_{\mathrm{f}} & =\frac{2(2.8 \mathrm{~m})}{1.5 \mathrm{~s}}-2.0 \mathrm{~m} / \mathrm{s} \\
& =1.7 \mathrm{~m} / \mathrm{s} \text { up the slope }
\end{aligned}
$$

11. 

| $\vec{v}_{0}$ | $\vec{v}_{\mathrm{f}}$ | $\vec{a}$ | $\vec{d}$ | $t$ |
| :---: | :---: | :---: | :---: | :---: |
| $?$ | $\times$ | $-9.81 \mathrm{~m} / \mathrm{s}^{2}$ | 5.0 m | 3.0 s |

$$
\begin{aligned}
\vec{d} & =\vec{v}_{0} t+\frac{1}{2} \vec{a} t^{2} \\
5.0 \mathrm{~m} & =\vec{v}_{0}(3.0 \mathrm{~s})+\frac{1}{2}\left(-9.81 \mathrm{~m} / \mathrm{s}^{2}\right)(3.0 \mathrm{~s})^{2} \\
\vec{v}_{0} & =\frac{5.0 \mathrm{~m}-\frac{1}{2}\left(-9.81 \mathrm{~m} / \mathrm{s}^{2}\right)(3.0 \mathrm{~s})^{2}}{3.0 \mathrm{~s}} \\
& =16 \mathrm{~m} / \mathrm{s} \mathrm{up}
\end{aligned}
$$

12. 

| $\vec{v}_{0}$ | $\vec{v}_{\mathrm{f}}$ | $\vec{a}$ | $\vec{d}$ | $t$ |
| :---: | :---: | :---: | :---: | :---: |
| $2.2 \mathrm{~m} / \mathrm{s}$ | $-1.1 \mathrm{~m} / \mathrm{s}$ | $?$ | $\times$ | 2.0 s |

$$
\begin{aligned}
\vec{a} & =\frac{\vec{v}_{\mathrm{f}}-\vec{v}_{0}}{t} \\
& =\frac{-1.1 \mathrm{~m} / \mathrm{s}-2.2 \mathrm{~m} / \mathrm{s}}{2.0 \mathrm{~s}} \\
& =-1.7 \mathrm{~m} / \mathrm{s}^{2} \text { or } 1.7 \mathrm{~m} / \mathrm{s}^{2} \text { down the slope }
\end{aligned}
$$

13. 

| $\vec{v}_{0}$ | $\vec{v}_{\mathrm{f}}$ | $\vec{a}$ | $\vec{d}$ | $t$ |
| :---: | :---: | :---: | :---: | :---: |
| $2.5 \mathrm{~m} / \mathrm{s}$ | $-1.6 \mathrm{~m} / \mathrm{s}$ | $?$ | 1.0 m | $\times$ |

$$
\begin{aligned}
\vec{v}_{\mathrm{f}}^{2}= & \vec{v}_{0}^{2}+2 \vec{a} \vec{d} \\
(-1.6 \mathrm{~m} / \mathrm{s})^{2} & =(2.5 \mathrm{~m} / \mathrm{s})^{2}+2 \vec{a}(1.0 \mathrm{~m}) \\
\vec{a}= & \frac{(-1.6 \mathrm{~m} / \mathrm{s})^{2}-(2.5 \mathrm{~m} / \mathrm{s})^{2}}{2(1.0 \mathrm{~m})} \\
= & -1.8 \mathrm{~m} / \mathrm{s}^{2} \\
& \text { or } 1.8 \mathrm{~m} / \mathrm{s}^{2} \text { down the slope }
\end{aligned}
$$

14. 

| $\vec{v}_{0}$ | $\vec{v}_{\mathrm{f}}$ | $\vec{a}$ | $\vec{d}$ | $t$ |
| :---: | :---: | :---: | :---: | :---: |
| $2.0 \mathrm{~m} / \mathrm{s}$ | 0 | $\times$ | 2.7 m | $?$ |

$$
\begin{aligned}
\vec{d} & =\left(\frac{\vec{v}_{\mathrm{f}}+\vec{v}_{0}}{2}\right) t \\
2.7 \mathrm{~m} & =\left(\frac{0+2.0 \mathrm{~m} / \mathrm{s}}{2}\right) t \\
t & =\frac{2(2.7 \mathrm{~m})}{2.0 \mathrm{~m} / \mathrm{s}} \\
& =2.7 \mathrm{~s}
\end{aligned}
$$

This is the time for the ball to roll up the incline. It will take the same time to come down. Therefore, the total time is:

$$
\begin{aligned}
t & =2(2.7 \mathrm{~s}) \\
& =5.4 \mathrm{~s}
\end{aligned}
$$

15. 

| $\vec{v}_{0}$ | $\vec{v}_{\mathrm{f}}$ | $\vec{a}$ | $\vec{d}$ | $t$ |
| :---: | :---: | :---: | :---: | :---: |
| $5.0 \mathrm{~m} / \mathrm{s}$ | $-5.0 \mathrm{~m} / \mathrm{s}$ | $?$ | 0 | 3.0 s |

$\vec{a}=\frac{\vec{v}_{\mathrm{f}}-\vec{v}_{0}}{t}$
$=\frac{(-5.0 \mathrm{~m} / \mathrm{s})-5.0 \mathrm{~m} / \mathrm{s}}{3.0 \mathrm{~s}}$
$=-3.3 \mathrm{~m} / \mathrm{s}^{2}$ or $3.3 \mathrm{~m} / \mathrm{s}^{2}$ down
16.

| $\vec{v}_{0}$ | $\vec{v}_{\mathrm{f}}$ | $\vec{a}$ | $\vec{d}$ | $t$ |
| :---: | :---: | :---: | :---: | :---: |
| $20.0 \mathrm{~m} / \mathrm{s}$ | $?$ | $-9.81 \mathrm{~m} / \mathrm{s}^{2}$ | -30.0 m | $?$ |

a) $\quad \vec{v}_{\mathrm{f}}^{2}=\vec{v}_{0}^{2}+2 \vec{a} \vec{d}$

$$
\begin{aligned}
& =(20.0 \mathrm{~m} / \mathrm{s})^{2}+2\left(-9.81 \mathrm{~m} / \mathrm{s}^{2}\right)(-30.0 \mathrm{~m}) \\
\stackrel{\rightharpoonup}{\mathrm{v}}_{\mathrm{f}} & =-31.4 \mathrm{~m} / \mathrm{s} \text { or } 31.4 \mathrm{~m} / \mathrm{s} \text { down }
\end{aligned}
$$

b) $\quad \vec{a}=\frac{\vec{v}_{\mathrm{f}}-\vec{v}_{0}}{t}$

$$
\begin{aligned}
-9.81 \mathrm{~m} / \mathrm{s}^{2} & =\frac{(-31.4-20.0) \mathrm{m} / \mathrm{s}}{t} \\
t & =\frac{(-31.4-20.0) \mathrm{m} / \mathrm{s}}{-9.81 \mathrm{~m} / \mathrm{s}^{2}} \\
t & =5.24 \mathrm{~s}
\end{aligned}
$$

17. 

| $\vec{v}_{0}$ | $\vec{v}_{\mathrm{f}}$ | $\vec{a}$ | $\vec{d}$ | $t$ |
| :---: | :---: | :---: | :---: | :---: |
| $5.0 \mathrm{~m} / \mathrm{s}$ | $\times$ | $?$ | 0 | 3.0 s |

$\vec{d}=\vec{v}_{0} t+\frac{1}{2} \vec{a} t^{2}$
$0=(5.0 \mathrm{~m} / \mathrm{s})(3.0 \mathrm{~s})+\frac{1}{2} \vec{a}(3.0 \mathrm{~s})^{2}$
$\vec{a}=-\frac{2(5.0 \mathrm{~m})}{(3.0 \mathrm{~s})}$
$\vec{a}=-3.3 \mathrm{~m} / \mathrm{s}^{2}$ or $3.3 \mathrm{~m} / \mathrm{s}^{2}$ down
18.

| $\vec{v}_{0}$ | $\vec{v}_{\mathrm{f}}$ | $\vec{a}$ | $\vec{d}$ | $t$ |
| :---: | :---: | :---: | :---: | :---: |
| $11.0 \mathrm{~m} / \mathrm{s}$ | $\times$ | $-9.81 \mathrm{~m} / \mathrm{s}^{2}$ | -5.0 m | $?$ |

First, find the final velocity:

$$
\begin{aligned}
\stackrel{\rightharpoonup}{\mathrm{f}}_{\mathrm{f}}^{2} & =\vec{v}_{0}^{2}+2 \vec{a} \vec{d} \\
& =(11.0 \mathrm{~m} / \mathrm{s})^{2}+2\left(-9.81 \mathrm{~m} / \mathrm{s}^{2}\right)(-5.0 \mathrm{~m}) \\
\stackrel{v}{\mathrm{f}} & =-14.8 \mathrm{~m} / \mathrm{s} \text { or } 14.8 \mathrm{~m} / \mathrm{s} \text { down }
\end{aligned}
$$

Now find the time:

$$
\begin{aligned}
\vec{a} & =\frac{\vec{v}_{\mathrm{f}}-\vec{v}_{0}}{t} \\
-9.81 \mathrm{~m} / \mathrm{s}^{2} & =\frac{(-14.8 \mathrm{~m} / \mathrm{s})-11.0 \mathrm{~m} / \mathrm{s}}{t} \\
t & =\frac{(-14.8 \mathrm{~m} / \mathrm{s})-11.0 \mathrm{~m} / \mathrm{s}}{-9.81 \mathrm{~m} / \mathrm{s}^{2}} \\
& =2.6 \mathrm{~s}
\end{aligned}
$$

19. 

| $\vec{v}_{0}$ | $\vec{v}_{\mathrm{f}}$ | $\vec{a}$ | $\vec{d}$ | $t$ |
| :---: | :---: | :---: | :---: | :---: |
| $\times$ | 0 | $-9.81 \mathrm{~m} / \mathrm{s}^{2}$ | $?$ | 2.65 s |

If it takes 5.30 s to go up and down, it will take $\frac{5.30 \mathrm{~s}}{2}=2.65 \mathrm{~s}$ to reach the highest point.

Now find the final velocity:

$$
\begin{aligned}
\vec{a} & =\frac{\vec{v}_{\mathrm{f}}-\vec{v}_{0}}{t} \\
-9.81 \mathrm{~m} / \mathrm{s}^{2} & =\frac{0-\vec{v}_{0}}{2.65 \mathrm{~s}} \\
\vec{v}_{0} & =26.0 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

Find the displacement:

$$
\begin{aligned}
\vec{d} & =\left(\frac{\vec{v}_{\mathrm{f}}+\vec{v}_{0}}{2}\right) t \\
& =\left(\frac{0+26.0 \mathrm{~m} / \mathrm{s}}{2}\right)(2.65 \mathrm{~s}) \\
& =34.4 \mathrm{~m}
\end{aligned}
$$

20. 

| $\vec{v}_{0}$ | $\vec{v}_{\mathrm{f}}$ | $\vec{a}$ | $\vec{d}$ | $t$ |
| :---: | :---: | :---: | :---: | :---: |
| 0 | $?$ | $-9.81 \mathrm{~m} / \mathrm{s}^{2}$ | -25.0 m | $?$ |

Either find the final velocity or time:
$\vec{v}_{\mathrm{f}}^{2}=\vec{v}_{0}^{2}+2 \vec{a} \vec{d}$
$=2\left(-9.81 \mathrm{~m} / \mathrm{s}^{2}\right)(-25.0 \mathrm{~m})$
$\vec{v}_{\mathrm{f}}=-22.1 \mathrm{~m} / \mathrm{s}$ or $22.1 \mathrm{~m} / \mathrm{s}$ down
$\vec{v}_{\mathrm{av}}=\frac{\vec{v}_{\mathrm{f}}+\vec{v}_{0}}{2}$
$=\frac{(-22.1 \mathrm{~m} / \mathrm{s})-0}{2}$
$=-11.1 \mathrm{~m} / \mathrm{s}$ or $11.1 \mathrm{~m} / \mathrm{s}$ down
21.

| $\vec{v}_{0}$ | $\vec{v}_{\mathrm{f}}$ | $\vec{a}$ | $\vec{d}$ | $t$ |
| :---: | :---: | :---: | :---: | :---: |
| $14.0 \mathrm{~m} / \mathrm{s}$ | 0 | $-9.81 \mathrm{~m} / \mathrm{s}^{2}$ | $?$ | $?$ |

a) $\quad \vec{v}_{\mathrm{f}}^{2}=\vec{v}_{0}^{2}+2 \vec{a} \vec{d}$

$$
\begin{aligned}
0 & =(14.0 \mathrm{~m} / \mathrm{s})^{2}+2\left(-9.81 \mathrm{~m} / \mathrm{s}^{2}\right) \vec{d} \\
\vec{d} & =\frac{-(14.0 \mathrm{~m} / \mathrm{s})^{2}}{2\left(-9.81 \mathrm{~m} / \mathrm{s}^{2}\right)} \\
& =9.99 \mathrm{~m} \mathrm{up}
\end{aligned}
$$

b) $\quad \vec{a}=\frac{\vec{v}_{\mathrm{f}}-\vec{v}_{0}}{t}$

$$
-9.81 \mathrm{~m} / \mathrm{s}^{2}=\frac{0-14.0 \mathrm{~m} / \mathrm{s}}{t}
$$

$$
t=1.43 \mathrm{~s}
$$

22. 

| $\vec{v}_{0}$ | $\vec{v}_{\mathrm{f}}$ | $\vec{a}$ | $\vec{d}$ | $t$ |
| :---: | :---: | :---: | :---: | :---: |
| $\times$ | $-1.90 \mathrm{~m} / \mathrm{s}$ | $?$ | 2.75 m | 4.50 s |

Find initial velocity first:

$$
\begin{aligned}
\vec{d} & =\left(\frac{\vec{v}_{\mathrm{f}}+\vec{v}_{0}}{2}\right) t \\
2.75 \mathrm{~m} & =\left(\frac{(-1.90 \mathrm{~m} / \mathrm{s})+\vec{v}_{0}}{2}\right)(4.50 \mathrm{~s}) \\
\vec{v}_{0} & =\frac{2(2.75 \mathrm{~m})}{4.50 \mathrm{~s}}+1.90 \mathrm{~m} / \mathrm{s} \\
& =3.12 \mathrm{~m} / \mathrm{s} \text { up the incline }
\end{aligned}
$$

Now find the acceleration:

$$
\begin{aligned}
\vec{a} & =\frac{\bar{v}_{\mathrm{f}}-\bar{v}_{0}}{t} \\
& =\frac{(-1.90 \mathrm{~m} / \mathrm{s})-3.12 \mathrm{~m} / \mathrm{s}}{4.50 \mathrm{~s}} \\
& =-1.12 \mathrm{~m} / \mathrm{s}^{2} \text { or } 1.12 \mathrm{~m} / \mathrm{s}^{2} \text { down the incline }
\end{aligned}
$$

## Lesson 7-Projectile Motion

## PRACTICE EXERCISES ANSWERS AND SOLUTIONS

1. Find the horizontal component of displacement.

Find $t$ from vertical component:

| $\vec{v}_{y 0}$ | $\vec{v}_{y f}$ | $\vec{a}_{y}$ | $\vec{d}_{y}$ | $t$ |
| :---: | :---: | :---: | :---: | :---: |
| 0 | $\times$ | $-9.81 \mathrm{~m} / \mathrm{s}^{2}$ | -90.0 m | $?$ |

$$
\begin{aligned}
\vec{d}_{y} & =\vec{v}_{y 0} t+\frac{1}{2} \vec{a}_{y} t^{2} \\
-90.0 \mathrm{~m} & =\frac{1}{2}\left(-9.81 \mathrm{~m} / \mathrm{s}^{2}\right) t^{2} \\
t & =\sqrt{\frac{2(-90.0 \mathrm{~m})}{-9.81 \mathrm{~m} / \mathrm{s}^{2}}} \\
& =4.28 \mathrm{~s}
\end{aligned}
$$

Now use $t$ to find the horizontal component of displacement:

$$
\begin{aligned}
\vec{v}_{x} & =\frac{\bar{d}_{x}}{t} \\
\vec{d}_{x} & =\vec{v}_{x} t \\
& =(10.0 \mathrm{~m} / \mathrm{s})(4.28 \mathrm{~s}) \\
& =42.8 \mathrm{~m}
\end{aligned}
$$

2. Find the horizontal component of displacement.

Find $t$ from vertical component:

| $\vec{v}_{y 0}$ | $\vec{v}_{y f}$ | $\vec{a}_{y}$ | $\vec{d}_{y}$ | $t$ |
| :---: | :---: | :---: | :---: | :---: |
| 0 | $\times$ | $-9.81 \mathrm{~m} / \mathrm{s}^{2}$ | $-1.50 \times 10^{2} \mathrm{~m}$ | $?$ |

$$
\begin{aligned}
\vec{d}_{y} & =\vec{v}_{y 0} t+\frac{1}{2} \vec{a}_{y} t^{2} \\
-1.50 \times 10^{2} \mathrm{~m} & =\frac{1}{2}\left(-9.81 \mathrm{~m} / \mathrm{s}^{2}\right) t^{2} \\
t & =\sqrt{\frac{2\left(-1.50 \times 10^{2} \mathrm{~m}\right)}{-9.81 \mathrm{~m} / \mathrm{s}^{2}}} \\
& =5.53 \mathrm{~s}
\end{aligned}
$$

Now use $t$ to find the horizontal component of displacement:

$$
\begin{aligned}
\stackrel{\rightharpoonup}{v}_{x} & =\frac{\vec{d}_{x}}{t} \\
\vec{d}_{x} & =\vec{v}_{x} t \\
& =(25.0 \mathrm{~m} / \mathrm{s})(5.53 \mathrm{~s}) \\
& =1.38 \times 10^{2} \mathrm{~m}
\end{aligned}
$$

3. Find the vertical component of displacement.

Find $t$ from the horizontal component:

$$
\begin{aligned}
\vec{v}_{x} & =\frac{\vec{d}_{x}}{t} \\
t & =\frac{\vec{d}_{x}}{\vec{v}_{x}} \\
& =\frac{100.0 \mathrm{~m}}{18.0 \mathrm{~m} / \mathrm{s}} \\
& =5.56 \mathrm{~s}
\end{aligned}
$$

Now use $t$ to find the vertical component of displacement:

| $\vec{v}_{y 0}$ | $\vec{v}_{y f}$ | $\vec{a}_{y}$ | $\vec{d}_{y}$ | $t$ |
| :---: | :---: | :---: | :---: | :---: |
| 0 | $\times$ | $-9.81 \mathrm{~m} / \mathrm{s}^{2}$ | $?$ | 5.56 s |

$$
\begin{aligned}
\stackrel{\rightharpoonup}{d}_{y} & =\vec{v}_{y 0} t+\frac{1}{2} \vec{a}_{y} t^{2} \\
& =\frac{1}{2}\left(-9.81 \mathrm{~m} / \mathrm{s}^{2}\right)(5.56 \mathrm{~s})^{2} \\
& =-152 \mathrm{~m}
\end{aligned}
$$

$\therefore$ the height of the cliff is 152 m .
4. Find the vertical component of displacement.

Find $t$ from the horizontal component:

$$
\begin{aligned}
\vec{v}_{x} & =\frac{\vec{d}_{x}}{t} \\
t & =\frac{\stackrel{\rightharpoonup}{d}_{x}}{\vec{v}_{x}} \\
& =\frac{48.0 \mathrm{~m}}{20.0 \mathrm{~m} / \mathrm{s}} \\
& =2.40 \mathrm{~s}
\end{aligned}
$$

Now use $t$ to find the vertical component of displacement:

| $\vec{v}_{y 0}$ | $\vec{v}_{y f}$ | $\vec{a}_{y}$ | $\vec{d}_{y}$ | $t$ |
| :---: | :---: | :---: | :---: | :---: |
| 0 | $\times$ | $-9.81 \mathrm{~m} / \mathrm{s}^{2}$ | $?$ | 2.40 s |

$$
\begin{aligned}
\stackrel{\rightharpoonup}{d}_{y} & =\vec{v}_{y 0} t+\frac{1}{2} \vec{a}_{y} t^{2} \\
& =\frac{1}{2}\left(-9.81 \mathrm{~m} / \mathrm{s}^{2}\right)(2.40 \mathrm{~s})^{2} \\
& =-28.3 \mathrm{~m}
\end{aligned}
$$

$\therefore$ the height of the cliff is 28.3 m .
5. Find the vertical component of displacement.

| $\vec{v}_{y 0}$ | $\vec{v}_{y f}$ | $\vec{a}_{y}$ | $\vec{d}_{y}$ | $t$ |
| :---: | :---: | :---: | :---: | :---: |
| 0 | $\times$ | $-9.81 \mathrm{~m} / \mathrm{s}^{2}$ | $?$ | 5.50 s |

$$
\begin{aligned}
\vec{d}_{y} & =\vec{v}_{y 0} t+\frac{1}{2} \vec{a}_{y} t^{2} \\
& =\frac{1}{2}\left(-9.81 \mathrm{~m} / \mathrm{s}^{2}\right)(5.50 \mathrm{~s})^{2} \\
& =-148 \mathrm{~m}
\end{aligned}
$$

$\therefore$ the height of the building is 148 m .
6. Find the horizontal component of displacement.

$$
\begin{aligned}
\stackrel{\rightharpoonup}{v}_{x} & =\frac{\vec{d}_{x}}{t} \\
\vec{d}_{x} & =\vec{v}_{x} t \\
& =(20.0 \mathrm{~m} / \mathrm{s})(4.20 \mathrm{~s}) \\
& =84.0 \mathrm{~m}
\end{aligned}
$$

7. a)

| Time (s) | Displacement from $t=0$ | $\begin{gathered} \text { Displacement } \\ \text { during time } \\ \text { interval } \\ \left(\times 10^{\mathbf{3}} \mathbf{m}\right) \end{gathered}$ |  | Average velocity during time interval ( $\times 10^{2} \mathrm{~m} / \mathrm{s}$ ) |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | horiz. | vert. | horiz. | vert. |
| 0.10 | 0.5 | 5.0 | 1.5 | 5.0 | 1.5 |
| 0.20 | 1.0 | 5.0 | 1.5 | 5.0 | 1.5 |
| 0.30 | 1.5 | 5.0 | 4.0 | 5.0 | 4.0 |
| 0.40 | 2.0 | 5.0 | 4.0 | 5.0 | 4.0 |
| 0.50 | 2.5 | 5.0 | 5.0 | 5.0 | 5.0 |
| 0.60 | 3.0 | 5.0 | 6.0 | 5.0 | 6.0 |
| 0.70 | 3.5 | 5.0 | 7.0 | 5.0 | 7.0 |
| 0.80 | 4.0 | 5.0 | 8.0 | 5.0 | 8.0 |
| 0.90 | 4.5 | 5.0 | 9.0 | 5.0 | 9.0 |
| 1.00 | 5.0 | 5.0 | 10.0 | 5.0 | 10.0 |
| 1.10 | 5.5 | 5.0 | 11.0 | 5.0 | 11.0 |

b)

c)

d) i) slope $=0$
$\therefore$ acceleration $=0$
ii) slope $=\frac{\text { rise }}{\text { run }}$

$$
\begin{aligned}
& =\frac{(0.10-0.01) \mathrm{m} / \mathrm{s}}{(0.95-0.05) \mathrm{s}} \\
\vec{a}_{\mathrm{y}} & =0.10 \mathrm{~m} / \mathrm{s}^{2} \text { or } 0.10 \mathrm{~m} / \mathrm{s}^{2} \text { down }
\end{aligned}
$$

8. You are asked to find the horizontal component of velocity.

Find $t$ from the vertical component

| $\vec{v}_{y 0}$ | $\vec{v}_{y f}$ | $\vec{a}_{y}$ | $\vec{d}_{y}$ | $t$ |
| :---: | :---: | :---: | :---: | :---: |
| 0 | $\times$ | $-9.81 \mathrm{~m} / \mathrm{s}^{2}$ | -85.0 m | $?$ |

$$
\begin{aligned}
\vec{d}_{y} & =\vec{v}_{y 0} t+\frac{1}{2} \vec{a}_{y} t^{2} \\
-85.0 \mathrm{~m} & =\frac{1}{2}\left(-9.81 \mathrm{~m} / \mathrm{s}^{2}\right) t^{2} \\
t & =4.16 \mathrm{~s}
\end{aligned}
$$

Now use $t$ to find the vertical component of velocity:

$$
\begin{aligned}
\stackrel{\rightharpoonup}{v}_{x} & =\frac{\vec{d}_{x}}{t} \\
& =\frac{67.8 \mathrm{~m}}{4.16 \mathrm{~s}} \\
& =16.3 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

## Practice Test

## ANSWERS AND SOLUTIONS

1. The motion of a falling object is uniform accelerated motion. This means that the acceleration of a falling object remains constant. Remember, the slope of a velocity-time graph is the acceleration. All of A., B., and D., represent a constant slope; but A. shows no change in velocity; $D$. shows that the velocity is decreasing.

B is the answer.
2. The slope of a displacement-time graph represents the velocity. For a falling object, the velocity increases. Therefore you are looking for a graph that shows an increasing slope.
$\mathbf{C}$ is the answer.
3. The acceleration of a falling object is constant. Therefore you are looking for a graph that shows a constant acceleration.

A is the answer.
4. Instantaneous speed is the speed at any instant in time. Average speed is a measure of distance travelled divided by the time it took to travel.
5. Velocity is slope of a position-time graph

$$
\begin{aligned}
\text { slope } & =\frac{\text { rise }}{\text { run }} \\
& =\frac{50 \mathrm{~m}-20 \mathrm{~m}}{6 \mathrm{~s}} \\
\vec{v} & =5.0 \mathrm{~m} / \mathrm{s} \mathrm{right}
\end{aligned}
$$

$\mathbf{A}$ is the answer.
6. In uniform motion the acceleration of an object is zero. The velocity of an object remains constant and moves in a straight line. The distances changes at a linear (constant) rate along that line.
7. The magnitude of the velocity will be at a maximum at the height it was thrown and zero at maximum height. The direction of the vector will be up as it moves toward its maximum height. It will have no direction at the moment the velocity is zero. The direction of the velocity vector will be down as the ball falls back towards the player.
8. Displacement is the area under a velocity-time graph.

$$
\begin{aligned}
\text { area } & =\frac{1}{2}(l \times w)+(l \times w) \\
& =\frac{1}{2}(30 \mathrm{~m} / \mathrm{s} \times 6.0 \mathrm{~s})+(20 \mathrm{~m} / \mathrm{s} \times 6.0 \mathrm{~s}) \\
& =90 \mathrm{~m}+120 \mathrm{~m} \\
& =210 \mathrm{~m} \text { down }
\end{aligned}
$$

D is the answer.
9. The slope of a position-time graph is the velocity. In this graph, the slope is decreasing. Therefore, the velocity is decreasing.
10. a) The velocity of the ball will be $-8.0 \mathrm{~m} / \mathrm{s}$ or $8.0 \mathrm{~m} / \mathrm{s}$ down.
b) The acceleration will be $-9.81 \mathrm{~m} / \mathrm{s}^{2}$ or $9.81 \mathrm{~m} / \mathrm{s}^{2}$ down due to gravity.
c) The acceleration will be $-9.81 \mathrm{~m} / \mathrm{s}^{2}$ or $9.81 \mathrm{~m} / \mathrm{s}^{2}$ down due to gravity.
11. The magnitude of the displacement vector will increase as it moves up the ramp. As the ball rolls back down, it will decrease but remain positive until reaching the point that it was released where it will equal zero. As it rolls further, the displacement will become negative and continue to decrease as it rolls down. The direction will always point up as the ball reaches its maximum height and then falls back down to the point of release. After falling below the point of the release, the displacement vector will always point downward.
12.

| $\vec{v}_{0}$ | $\vec{v}_{\mathrm{f}}$ | $\vec{a}$ | $\vec{d}$ | $t$ |
| :---: | :---: | :---: | :---: | :---: |
| $2.2 \mathrm{~m} / \mathrm{s}$ | 0 | $\times$ | 3.2 m | $?$ |

$$
\begin{aligned}
\vec{d} & =\left(\frac{\vec{v}_{\mathrm{f}}+\vec{v}_{0}}{2}\right) t \\
3.2 \mathrm{~m} & =\left(\frac{0+2.2 \mathrm{~m} / \mathrm{s}}{2}\right) t \\
3.2 \mathrm{~m} & =(1.1 \mathrm{~m} / \mathrm{s}) t \\
t & =2.9 \mathrm{~s}
\end{aligned}
$$

$\mathbf{B}$ is the answer.

13

| $\vec{v}_{0}$ | $\vec{v}_{\mathrm{f}}$ | $\vec{a}$ | $\vec{d}$ | $t$ |
| :---: | :---: | :---: | :---: | :---: |
| $15 \mathrm{~m} / \mathrm{s}$ | $\times$ | $-9.81 \mathrm{~m} / \mathrm{s}^{2}$ | $?$ | 8.00 s |

$$
\begin{aligned}
\vec{d} & =\vec{v}_{0} t+\frac{1}{2} \vec{a} t^{2} \\
& =(15 \mathrm{~m} / \mathrm{s})(8.00 \mathrm{~s})+\frac{1}{2}\left(-9.81 \mathrm{~m} / \mathrm{s}^{2}\right)(8.00)^{2} \\
& =-1.9 \times 10^{2} \mathrm{~m} \text { or } 1.9 \times 10^{2} \mathrm{~m} \text { down }
\end{aligned}
$$

The height is the magnitude of the displacement.
$\therefore d=1.9 \times 10^{2} \mathrm{~m}$
$\mathbf{B}$ is the answer.
14.

| $\vec{v}_{0}$ | $\vec{v}_{\mathrm{f}}$ | $\vec{a}$ | $\vec{d}$ | $t$ |
| :---: | :---: | :---: | :---: | :---: |
| $12 \mathrm{~m} / \mathrm{s}$ | $?$ | $-9.81 \mathrm{~m} / \mathrm{s}^{2}$ | $\times$ | 8.0 s |

$$
\begin{aligned}
\vec{a} & =\frac{\vec{v}_{\mathrm{f}}-\vec{v}_{0}}{t} \\
-9.81 \mathrm{~m} / \mathrm{s}^{2} & =\frac{\vec{v}_{\mathrm{f}}-12 \mathrm{~m} / \mathrm{s}}{8.0 \mathrm{~s}} \\
\vec{v}_{\mathrm{f}} & =-66 \mathrm{~m} / \mathrm{s} \text { or } 66 \mathrm{~m} / \mathrm{s} \text { down }
\end{aligned}
$$

Speed is the magnitude of the velocity.
$\mathbf{B}$ is the answer.
15. a) The objects hit the ground at the same velocity.
b) The object thrown upward will reach a velocity of zero due to the downward acceleration of gravity at some point above the student. It will then fall until it reaches the height that it was thrown. At this point, its velocity will be $5.0 \mathrm{~m} / \mathrm{s}$ down because the acceleration is constant. This is the same velocity that the second object is thrown downward. Since both objects have the same velocity at the same height, they will hit the ground with the same velocity.
16. All objects fall at the same rate if you ignore air friction. Also, the vertical component of the motion is independent of the horizontal component.
$\mathbf{C}$ is the answer.
17. All objects fall at the same rate if you ignore air friction. Also, the vertical component of the motion is independent of the horizontal component.
$\mathbf{C}$ is the answer.
18. The vertical component's acceleration will be constant and downward due to gravity. The initial velocity is positive but will decrease until the rock reaches its maximum height where it will be zero. The velocity will remain negative and continue to decrease below zero until it reaches maximum velocity when it hits the ground. The horizontal component will have an acceleration of zero. Its velocity will remain constant throughout its flight.
19. All objects fall at the same rate. Also the vertical component of the motion is independent of the horizontal component. Therefore, the lower the cliff, the sooner the object will hit the ground.
$\mathbf{C}$ is the answer.
20. The slope of a position-time graph represents the velocity of the object. Therefore, a steeper slope will represent a greater velocity.
$\mathbf{A}$ is the answer.
21. They all have zero acceleration. Slope of a position-time graph represents the velocity. All these graphs have a constant slope. Therefore, they have a constant velocity. If the velocity is constant, there is no acceleration.
22. The slope of a velocity-time graph represents the velocity. Therefore, the greatest constant acceleration is the section that has the greatest constant slope.
$\mathbf{B}$ is the answer.
23. Sections A, B, C have a constant slope, which means that in these sections they have a constant velocity. At any instant in time, the steepest slope can be found at the end of section $D$.

D is the answer.
24. The acceleration due to gravity is $-9.81 \mathrm{~m} / \mathrm{s}^{2}$ for all objects near the surface of the Earth if air friction is ignored; therefore it does not matter if the object is moving up or down, its acceleration is $-9.81 \mathrm{~m} / \mathrm{s}^{2}$.

D is the answer.
25. The object falls to the ground due to a change in velocity as it increases downward.
The acceleration of the object is constant due to gravity.

## FORCES

Lesson 1-Forces and Newton's Laws of Motion

## PRACTICE EXERCISES

 ANSWERS AND SOLUTIONS1. $\vec{F}_{\text {net }}=m \vec{a}$

$$
\begin{aligned}
\vec{a} & =\frac{\stackrel{\rightharpoonup}{F}_{\text {net }}}{m} \\
& =\frac{9.0 \mathrm{~N}}{20.0 \mathrm{~kg}} \\
& =0.45 \mathrm{~m} / \mathrm{s}^{2} \text { right }
\end{aligned}
$$

2. $\vec{F}_{\text {net }}=m \vec{a}$

$$
\begin{aligned}
m & =\frac{\vec{F}_{\text {net }}}{\vec{a}} \\
& =\frac{15.0 \mathrm{~N}}{8.0 \mathrm{~m} / \mathrm{s}^{2}} \\
& =1.9 \mathrm{~kg}
\end{aligned}
$$

3. $\vec{F}_{\text {net }}=m \vec{a}$

$$
\begin{aligned}
& =(16.0 \mathrm{~kg})\left(-2.0 \mathrm{~m} / \mathrm{s}^{2}\right) \\
& =-32 \mathrm{~N} \text { or } 32 \mathrm{~N} \text { left }
\end{aligned}
$$

4. $\vec{F}_{\text {net }}=m \vec{a}$

$$
\begin{aligned}
\vec{a} & =\frac{\stackrel{\rightharpoonup}{F}_{\text {net }}}{m} \\
& =\frac{10.2 \mathrm{~N}}{12.0 \mathrm{~kg}} \\
& =0.850 \mathrm{~m} / \mathrm{s}^{2} \text { right }
\end{aligned}
$$

5. $\vec{F}_{\text {net }}=m \vec{a}$

$$
\begin{aligned}
& =(5.2 \mathrm{~kg})\left(6.0 \mathrm{~m} / \mathrm{s}^{2}\right) \\
& =31 \mathrm{~N} \text { right }
\end{aligned}
$$

6. $\vec{F}_{\text {net }}=m \vec{a}$

$$
\begin{aligned}
\vec{a} & =\frac{\vec{F}_{\text {net }}}{m} \\
& =\frac{-2.0 \mathrm{~N}}{18 \mathrm{~kg}} \\
& =-0.11 \mathrm{~m} / \mathrm{s}^{2} \text { or } 0.11 \mathrm{~m} / \mathrm{s}^{2} \text { left }
\end{aligned}
$$

7. $\vec{a}=\frac{\vec{v}_{\mathrm{f}}-\vec{v}_{0}}{t}$

$$
\begin{aligned}
& =\frac{(-25.0 \mathrm{~m} / \mathrm{s})-0}{10.0 \mathrm{~s}} \\
& =-2.500 \mathrm{~m} / \mathrm{s}^{2} \text { or } 2.500 \mathrm{~m} / \mathrm{s}^{2} \text { left }
\end{aligned}
$$

$\vec{F}_{\text {net }}=m \vec{a}$
$=(925 \mathrm{~kg})\left(-2.500 \mathrm{~m} / \mathrm{s}^{2}\right)$
$=-2.31 \times 10^{3} \mathrm{~N}$ or $2.31 \times 10^{3} \mathrm{~N}$ left
8. $\vec{d}=\vec{v}_{0} t+\frac{1}{2} \vec{a} t^{2}$
$132 \mathrm{~m}=\frac{1}{2} \vec{a}(12.0 \mathrm{~s})^{2}$
$\vec{a}=1.833 \mathrm{~m} / \mathrm{s}^{2}$ right

$$
\begin{aligned}
\stackrel{\rightharpoonup}{F}_{\text {net }} & =m \vec{a} \\
& =\left(1.08 \times 10^{3} \mathrm{~kg}\right)\left(1.833 \mathrm{~m} / \mathrm{s}^{2}\right) \\
& =1.98 \times 10^{3} \mathrm{~N} \text { right }
\end{aligned}
$$

9. $\vec{v}_{f}^{2}=\bar{v}_{0}^{2}+2 \bar{a} \vec{d}$

$$
\begin{aligned}
(12 \mathrm{~m} / \mathrm{s})^{2} & =(5.0 \mathrm{~m} / \mathrm{s})^{2}+2 \bar{a}(94 \mathrm{~m}) \\
\vec{a} & =0.633 \mathrm{~m} / \mathrm{s}^{2} \text { right }
\end{aligned}
$$

$$
\begin{aligned}
\vec{F}_{\text {net }} & =m \vec{a} \\
& =\left(1.20 \times 10^{3} \mathrm{~kg}\right)\left(0.633 \mathrm{~m} / \mathrm{s}^{2}\right) \\
& =7.6 \times 10^{2} \mathrm{Nright}
\end{aligned}
$$

10. Convert $48 \mathrm{~km} / \mathrm{h}$ to $\mathrm{m} / \mathrm{s}$ :

$$
\begin{aligned}
\stackrel{\rightharpoonup}{\mathrm{v}}_{\mathrm{f}} & =48 \mathrm{~km} / \mathrm{h} \times 1000 \mathrm{~m} / \mathrm{km} \times \frac{1 \mathrm{~h}}{3600 \mathrm{~s}} \\
& =13.3 \mathrm{~m} / \mathrm{s} \\
\vec{a} & =\frac{\bar{v}_{\mathrm{f}}-\vec{v}_{0}}{t} \\
& =\frac{13.3 \mathrm{~m} / \mathrm{s}-0}{5.0 \mathrm{~s}} \\
& =2.66 \mathrm{~m} / \mathrm{s}^{2} \mathrm{right}
\end{aligned}
$$

$$
\begin{aligned}
\stackrel{\rightharpoonup}{F}_{\text {net }} & =m \vec{a} \\
m & =\frac{\vec{F}_{\text {net }}}{\vec{a}} \\
& =\frac{2.5 \times 10^{3} \mathrm{~N}}{2.66 \mathrm{~m} / \mathrm{s}^{2}} \\
& =9.4 \times 10^{2} \mathrm{~kg}
\end{aligned}
$$

11. a) $\vec{F}_{\text {net }}=m \vec{a}$

$$
\begin{aligned}
\vec{a} & =\frac{\vec{F}_{\text {net }}}{m} \\
& =\frac{6.6 \mathrm{~N}}{9.0 \mathrm{~kg}} \\
& =0.733 \mathrm{~m} / \mathrm{s}^{2} \text { right }
\end{aligned}
$$

$$
\begin{aligned}
\vec{v}_{f}^{2} & =\vec{v}_{0}^{2}+2 \vec{a} \vec{d} \\
(3.0 \mathrm{~m} / \mathrm{s})^{2} & =2\left(0.733 \mathrm{~m} / \mathrm{s}^{2}\right) \vec{d} \\
\vec{d} & =6.1 \mathrm{~m} \mathrm{right}
\end{aligned}
$$

b) $\quad \vec{a}=\frac{\vec{v}_{\mathrm{f}}-\vec{v}_{0}}{t}$

$$
\begin{aligned}
0.733 \mathrm{~m} / \mathrm{s}^{2} & =\frac{3.0 \mathrm{~m} / \mathrm{s}-0}{t} \\
t & =4.1 \mathrm{~s}
\end{aligned}
$$

## Lesson 2—Newton's Third Law of Motion

## PRACTICE EXERCISES ANSWERS AND SOLUTIONS

1. $\stackrel{\rightharpoonup}{F}_{1}=-\stackrel{\rightharpoonup}{F}_{2}$

$$
m_{\mathrm{A}} \bar{a}_{\mathrm{A}}=-m_{\mathrm{B}} \vec{a}_{\mathrm{B}}
$$

$$
(38.0 \mathrm{~kg})\left(0.60 \mathrm{~m} / \mathrm{s}^{2}\right)=-m_{\mathrm{B}}\left(-0.75 \mathrm{~m} / \mathrm{s}^{2}\right)
$$

$$
m_{\mathrm{B}}=3.0 \times 10^{1} \mathrm{~kg}
$$

2. $\bar{F}_{\text {net }}=m \vec{a}$

$$
\begin{aligned}
\vec{a} & =\frac{\vec{F}_{\text {net }}}{m} \\
& =\frac{-125 \mathrm{~N}}{50.0 \mathrm{~kg}} \\
& =-2.500 \mathrm{~m} / \mathrm{s}^{2} \text { or } 2.500 \mathrm{~m} / \mathrm{s}^{2} \mathrm{left}
\end{aligned}
$$

$$
\begin{aligned}
\vec{a} & =\frac{\vec{v}_{\mathrm{f}}-\vec{v}_{0}}{t} \\
-2.500 \mathrm{~m} / \mathrm{s}^{2} & =\frac{\vec{v}_{\mathrm{f}}-0}{0.110 \mathrm{~s}} \\
\vec{v}_{\mathrm{f}} & =-0.275 \mathrm{~m} / \mathrm{s} \text { or } 0.275 \mathrm{~m} / \mathrm{s} \text { left }
\end{aligned}
$$

3. $\vec{a}=\frac{\vec{v}_{\mathrm{f}}-\vec{v}_{0}}{t}$

$$
\begin{aligned}
& =\frac{22 \mathrm{~m} / \mathrm{s}-11 \mathrm{~m} / \mathrm{s}}{0.75 \mathrm{~s}} \\
& =14.7 \mathrm{~m} / \mathrm{s}^{2} \text { right }
\end{aligned}
$$

$$
\begin{aligned}
\vec{F} & =m \vec{a} \\
& =\left(9.8 \times 10^{3} \mathrm{~kg}\right)\left(14.7 \mathrm{~m} / \mathrm{s}^{2}\right) \\
& =1.4 \times 10^{5} \mathrm{~N} \text { right }
\end{aligned}
$$

4. First, find the force exerted on the object and the time it takes to throw the object:

$$
\begin{aligned}
& \vec{v}_{\mathrm{Of}}^{2}=\vec{v}_{\mathrm{OO}}^{2}+2 \vec{a}_{\mathrm{o}} \bar{d} \\
&\left(9.6 \mathrm{~m} / \mathrm{s}^{2}\right)=2 \bar{a}_{\mathrm{o}}(0.60 \mathrm{~m}) \\
& \vec{a}_{\mathrm{O}}=76.8 \mathrm{~m} / \mathrm{s}^{2} \mathrm{right} \\
& \vec{F}_{\mathrm{O}}= m_{\mathrm{o}} \vec{a}_{\mathrm{O}} \\
&=(3.0 \mathrm{~kg})\left(76.8 \mathrm{~m} / \mathrm{s}^{2}\right) \\
&=230 \mathrm{Nright}
\end{aligned}
$$

$$
\begin{aligned}
\vec{a}_{\mathrm{O}} & =\frac{\overrightarrow{\mathrm{v}}_{\mathrm{Of}}-\vec{v}_{\mathrm{O} 0}}{t} \\
76.8 \mathrm{~m} / \mathrm{s}^{2} & =\frac{9.6 \mathrm{~m} / \mathrm{s}-0}{t} \\
t & =0.125 \mathrm{~s}
\end{aligned}
$$

Now, consider the reaction force exerted on the student:
$\vec{F}_{\mathrm{G}}=m_{\mathrm{G}} \vec{a}_{\mathrm{G}}$
$\vec{a}_{\mathrm{G}}=\frac{\vec{F}_{\mathrm{G}}}{m_{\mathrm{G}}}$
since $\vec{F}_{\mathrm{O}}=-\vec{F}_{\mathrm{G}}$
$\vec{a}_{\mathrm{G}}=\frac{-230 \mathrm{~N}}{45 \mathrm{~kg}}$
$=-5.11 \mathrm{~m} / \mathrm{s}^{2}$ or $5.11 \mathrm{~m} / \mathrm{s}^{2}$ left

$$
\vec{a}=\frac{\vec{v}_{\mathrm{Gf}}-\vec{v}_{\mathrm{G} 0}}{t}
$$

$-5.11 \mathrm{~m} / \mathrm{s}^{2}=\frac{\vec{v}_{\text {Gf }}-0}{0.125 \mathrm{~s}}$
$\bar{v}_{\mathrm{Gf}}=-0.64 \mathrm{~m} / \mathrm{s}$ or $0.64 \mathrm{~m} / \mathrm{s}$ left

## Lesson 3-Force Due to Gravity

## PRACTICE EXERCISES ANSWERS AND SOLUTIONS

1. $\vec{F}_{\mathrm{g}}=m \bar{g}$

$$
\begin{aligned}
& =(25.0 \mathrm{~kg})\left(-9.81 \mathrm{~m} / \mathrm{s}^{2}\right) \\
& =-245 \mathrm{~N} \text { or } 245 \mathrm{~N} \text { down }
\end{aligned}
$$

2. $\vec{F}_{\mathrm{g}}=m \bar{g}$

$$
\begin{aligned}
m & =\frac{\bar{F}_{g}}{\bar{g}} \\
& =\frac{80.0 \mathrm{~N}}{9.81 \mathrm{~m} / \mathrm{s}^{2}} \\
& =8.15 \mathrm{~kg}
\end{aligned}
$$

3. $\bar{F}_{\mathrm{g}}=m \bar{g}$

$$
\begin{aligned}
\bar{g} & =\frac{\vec{F}_{g}}{m} \\
& =\frac{-36.0 \mathrm{~N}}{22.0 \mathrm{~kg}} \\
& =-1.64 \mathrm{~m} / \mathrm{s}^{2} \text { or } 1.64 \mathrm{~m} / \mathrm{s}^{2} \text { down }
\end{aligned}
$$

4. $\vec{F}_{\mathrm{g}}=m \bar{g}$

$$
\begin{aligned}
& =(72.0 \mathrm{~kg})\left(-9.81 \mathrm{~m} / \mathrm{s}^{2}\right) \\
& =-706 \mathrm{~N} \text { or } 706 \mathrm{~N} \text { down }
\end{aligned}
$$

5. $\vec{F}_{\mathrm{g}}=m \bar{g}$

$$
\begin{aligned}
m & =\frac{\bar{F}_{\mathrm{g}}}{\bar{g}} \\
& =\frac{-127 \mathrm{~N}}{-9.81 \mathrm{~m} / \mathrm{s}^{2}} \\
& =12.9 \mathrm{~kg}
\end{aligned}
$$

## Lesson 4-Contact Forces

## PRACTICE EXERCISES ANSWERS AND SOLUTIONS

1. $\vec{F}_{\mathrm{N}}=-m \bar{g}$

$$
\begin{aligned}
& =-(14.0 \mathrm{~kg})\left(-9.81 \mathrm{~m} / \mathrm{s}^{2}\right) \\
& =137 \mathrm{Nup}
\end{aligned}
$$

2. $\vec{F}_{\mathrm{N}}=-m \bar{g}$

$$
\begin{aligned}
& =-(9.6 \mathrm{~kg})\left(-9.81 \mathrm{~m} / \mathrm{s}^{2}\right) \\
& =94.2 \mathrm{~N} \mathrm{up}
\end{aligned}
$$

$$
\begin{aligned}
F_{\mathrm{f}} & =\mu_{\mathrm{k}} F_{\mathrm{N}} \\
& =(0.11)(94.2 \mathrm{~N}) \\
& =1.0 \times 10^{1} \mathrm{~N}
\end{aligned}
$$

3. $F_{\mathrm{f}}=\mu_{\mathrm{k}} F_{\mathrm{N}}$

$$
\begin{aligned}
\mu_{\mathrm{k}} & =\frac{F_{\mathrm{f}}}{F_{\mathrm{N}}} \\
& =\frac{3.0 \mathrm{~N}}{20.0 \mathrm{~N}} \\
& =0.15
\end{aligned}
$$

4. $\vec{F}_{\mathrm{N}}=-m \bar{g}$

$$
\begin{aligned}
& =-(16.2 \mathrm{~kg})\left(-9.81 \mathrm{~m} / \mathrm{s}^{2}\right) \\
& =159 \mathrm{Nup}
\end{aligned}
$$

5. $\quad \vec{F}_{\text {net }}=m \vec{a}$

$$
\begin{aligned}
& =(6.2 \mathrm{~kg})\left(1.1 \mathrm{~m} / \mathrm{s}^{2}\right) \\
& =6.8 \mathrm{~N} \text { right }
\end{aligned}
$$

The net force is the sum of all forces acting on an object. This can be written as:

$$
\begin{aligned}
\vec{F}_{\text {net }} & =\vec{T}+\vec{F}_{\mathrm{f}} \\
\vec{F}_{\mathrm{f}} & =\vec{F}_{\text {net }}-\vec{T} \\
& =6.8 \mathrm{~N}-22.0 \mathrm{~N} \\
& =-15.2 \mathrm{~N} \text { or } 15.2 \mathrm{~N} \text { left }
\end{aligned}
$$

The force of friction is negative as expected since it is in the opposite direction of motion.

The coefficient of friction can be found using:
$F_{\mathrm{f}}=\mu F_{\mathrm{N}}$
Remember that for this equation only the magnitude of the force of friction and normal force are used. First, find the normal force:
$\vec{F}_{\mathrm{N}}=-m \vec{g}$
$\stackrel{\rightharpoonup}{F}_{\mathrm{N}}=-(6.2 \mathrm{~kg})\left(-9.81 \mathrm{~m} / \mathrm{s}^{2}\right)$
$\vec{F}_{\mathrm{N}}=60.8 \mathrm{~N}$ up
$F_{\mathrm{f}}=\mu_{\mathrm{k}} F_{\mathrm{N}}$
$\mu_{\mathrm{k}}=\frac{F_{\mathrm{f}}}{F_{\mathrm{N}}}$
$=\frac{15.2 \mathrm{~N}}{60.8 \mathrm{~N}}$
$=0.25$
6. $\stackrel{\rightharpoonup}{F}_{\mathrm{s}}=-k \vec{x}$

$$
\begin{aligned}
& =-(20.0 \mathrm{~N} / \mathrm{m})(0.100 \mathrm{~m}) \\
& =-2.00 \mathrm{~N} \text { or } 2.00 \mathrm{~N} \mathrm{left}
\end{aligned}
$$

7. Since the direction of the spring force is left, choose it as negative.

$$
\begin{aligned}
\vec{F}_{\mathrm{s}} & =-k \vec{x} \\
\vec{x} & =-\frac{\vec{F}_{\mathrm{s}}}{k} \\
& =-\frac{(-2.0 \mathrm{~N})}{15 \mathrm{~N} / \mathrm{m}} \\
& =0.13 \mathrm{~m} \text { right }
\end{aligned}
$$

8. $\vec{F}_{\mathrm{s}}=-k \vec{x}$

$$
\begin{aligned}
k & =-\frac{\stackrel{\rightharpoonup}{F}_{\mathrm{s}}}{\vec{x}} \\
& =-\frac{(-1.2 \mathrm{~N})}{0.025 \mathrm{~m}} \\
& =48 \mathrm{~N} / \mathrm{m}
\end{aligned}
$$

9. Choose the direction of displacement as negative because it is down. This makes the value of the spring force positive because it must be in the opposite direction.

$$
\begin{aligned}
\vec{F}_{\mathrm{s}} & =-k \vec{x} \\
k & =-\frac{\vec{F}_{\mathrm{s}}}{\vec{x}} \\
& =-\frac{1.65 \mathrm{~N}}{(-0.110 \mathrm{~m})} \\
& =15 \mathrm{~N} / \mathrm{m}
\end{aligned}
$$

10. Choosing up as positive:

$$
\begin{aligned}
\vec{F}_{\mathrm{g}} & =m \vec{g} \\
& =(5.0 \mathrm{~kg})\left(-9.81 \mathrm{~m} / \mathrm{s}^{2}\right) \\
& =-49 \mathrm{~N} \text { or } 49 \mathrm{~N} \text { down }
\end{aligned}
$$

The direction of displacement is the same as the direction of the acceleration due to gravity. Therefore, displacement is negative while the spring force is positive.

$$
\begin{aligned}
\vec{F}_{\mathrm{s}} & =-k \vec{x} \\
k & =-\frac{\vec{F}_{\mathrm{s}}}{\vec{x}} \\
& =-\frac{49 \mathrm{~N}}{\left(-3.25 \times 10^{-2} \mathrm{~m}\right)} \\
& =1.5 \times 10^{3} \mathrm{~N} / \mathrm{m}
\end{aligned}
$$

11. $\vec{F}_{\mathrm{s}}=-k \vec{x}$

$$
\begin{aligned}
\bar{x} & =-\frac{\vec{F}_{\mathrm{s}}}{k} \\
& =-\frac{9.3 \mathrm{~N}}{25 \mathrm{~N} / \mathrm{m}} \\
& =-0.37 \mathrm{~m} \text { or } 0.37 \mathrm{~m} \text { down }
\end{aligned}
$$

12. Choose the direction of displacement as positive. This makes the value of the spring force negative because it must be in the opposite direction.

$$
\begin{aligned}
\vec{F}_{\mathrm{s}} & =-k \vec{x} \\
& =-(5.0 \mathrm{~N} / \mathrm{m})(0.080) \\
& =-0.40 \mathrm{~N} \text { or } 0.40 \mathrm{~N} \text { left } \\
\vec{F}_{\text {net }} & =m \vec{a} \\
\vec{a} & =\frac{\vec{F}}{m} \\
& =\frac{-0.40 \mathrm{~N}}{7.5 \times 10^{-2} \mathrm{~kg}} \\
& =-5.3 \mathrm{~m} / \mathrm{s}^{2} \text { or } 5.3 \mathrm{~m} / \mathrm{s}^{2} \text { left }
\end{aligned}
$$

## Lesson 6-Forces and Motion

## PRACTICE EXERCISES ANSWERS AND SOLUTIONS

1. Consider up to be the positive direction.


$$
\begin{aligned}
\sum \vec{F} & =\vec{F}_{\text {net }} \\
& =\vec{F}_{\mathrm{T}}+\vec{F}_{\mathrm{g}} \\
& =145 \mathrm{~N}+m \vec{g} \\
& =145 \mathrm{~N}+(11.0 \mathrm{~kg})\left(-9.81 \mathrm{~m} / \mathrm{s}^{2}\right) \\
& =37.09 \mathrm{~N} \\
\vec{F}_{\text {net }} & =m \bar{a} \\
a & =\frac{F_{\text {net }}}{m} \\
& =\frac{37.09 \mathrm{~N}}{11.0 \mathrm{~kg}} \\
& =3.37 \mathrm{~m} / \mathrm{s}^{2}
\end{aligned}
$$

2. 



Consider east (right) to be the positive direction.

$$
\begin{aligned}
\sum \vec{F} & =\vec{F}_{\text {net }} \\
& =\vec{F}_{\text {app }}+\vec{F}_{\mathrm{f}} \\
& =6.0 \mathrm{~N}+(-2.0 \mathrm{~N}) \\
& =4.0 \mathrm{~N} \\
\vec{F}_{\text {net }} & =m \bar{a} \\
\vec{a} & =\frac{\vec{F}_{\text {net }}}{m} \\
& =\frac{4.0 \mathrm{~N}}{12.0 \mathrm{~kg}} \\
& =0.33 \mathrm{~m} / \mathrm{s}^{2} \text { or } 0.33 \mathrm{~m} / \mathrm{s}^{2} \text { east }
\end{aligned}
$$

3. 



Consider right to be the positive direction in the diagram.

$$
\begin{aligned}
\sum \stackrel{\rightharpoonup}{F} & =\stackrel{\rightharpoonup}{n}_{\text {net }} \\
& =m \bar{a} \\
& =(20 \mathrm{~kg})\left(0.80 \mathrm{~m} / \mathrm{s}^{2}\right) \\
& =16.0 \mathrm{~N}
\end{aligned}
$$

$$
\begin{aligned}
\vec{F}_{\text {net }} & =\vec{F}_{\mathrm{T}}+\vec{F}_{\mathrm{f}} \\
\vec{F}_{\mathrm{f}} & =\vec{F}_{\text {en }}-\vec{F}_{\mathrm{T}} \\
& =16.0 \mathrm{~N}-27.0 \mathrm{~N} \\
& =-11.0 \mathrm{~N} \\
F_{\mathrm{f}} & =11 \mathrm{~N}
\end{aligned}
$$

4. Consider up as the positive direction.

$$
\begin{aligned}
\sum \vec{F} & =\vec{F}_{\text {net }} \\
& =m \vec{a} \\
& =(6.3 \mathrm{~kg})\left(0.45 \mathrm{~m} / \mathrm{s}^{2}\right) \\
& =2.8 \mathrm{~N} \\
\vec{F}_{\mathrm{g} p} & =m \vec{g} \\
& =(6.3 \mathrm{~kg})\left(-9.81 \mathrm{~m} / \mathrm{s}^{2}\right) \\
& =-61.8 \mathrm{~N} \\
\vec{F}_{\text {net }} & =\vec{F}_{\text {app }}+\vec{F}_{\mathrm{g}} \\
F_{\text {app }} & =\vec{F}_{\text {net }}-\vec{F}_{\mathrm{g}} \\
& =2.8 \mathrm{~N}-(-61.8 \mathrm{~N}) \\
& =65 \mathrm{~N}
\end{aligned}
$$

5. a) Consider up as the positive direction.


$$
\begin{aligned}
\vec{F}_{\text {net }} & =\vec{F}_{\mathrm{g}}+\vec{F}_{\mathrm{T}} \\
\stackrel{\rightharpoonup}{\mathrm{~T}}_{\mathrm{T}} & =\vec{F}_{\text {net }}-\vec{F}_{\mathrm{g}} \\
& =-1.260 \times 10^{3} \mathrm{~N}-\left(-1.177 \times 10^{4} \mathrm{~N}\right) \\
& =1.05 \times 10^{4} \mathrm{~N} \text { or } 1.05 \times 10^{4} \mathrm{Nup}
\end{aligned}
$$

b) Consider up as the positive direction.

$$
\begin{aligned}
\sum \stackrel{\rightharpoonup}{F} & =\stackrel{\rightharpoonup}{F}_{\text {net }} \\
& =m \bar{a} \\
& =\left(1.20 \times 10^{3} \mathrm{~kg}\right)\left(1.05 \mathrm{~m} / \mathrm{s}^{2}\right) \\
& =1.260 \times 10^{3} \mathrm{~N} \\
\stackrel{\rightharpoonup}{F}_{\text {net }} & =\vec{F}_{\mathrm{T}}+\stackrel{\rightharpoonup}{F}_{g} \\
\stackrel{\rightharpoonup}{F}_{\mathrm{T}} & =\stackrel{F}{\text { net }}^{\stackrel{\rightharpoonup}{F}_{\mathrm{g}}} \\
& =\left(1.260 \times 10^{3} \mathrm{~N}\right)-\left(-1.177 \times 10^{4} \mathrm{~N}\right) \\
& =1.30 \times 10^{4} \mathrm{~N}
\end{aligned}
$$

c) acceleration $=0 ; \therefore \sum \stackrel{\rightharpoonup}{F}=0$

Consider up as the positive direction.

$$
\sum \vec{F}=\vec{F}_{\text {net }}
$$

$$
=\vec{F}_{\mathrm{T}}+\vec{F}_{\mathrm{g}}
$$

$$
=0
$$

or
$\vec{F}_{T}=-\vec{F}_{\mathrm{g}}$
$=-m \vec{g}$
$=-\left(1.20 \times 10^{3} \mathrm{~kg}\right)\left(-9.81 \mathrm{~m} / \mathrm{s}^{2}\right)$
$=1.18 \times 10^{4} \mathrm{~N}$
6. Assume that the direction of motion is the positive direction.

$$
\begin{aligned}
\sum \stackrel{\rightharpoonup}{F} & =\stackrel{\rightharpoonup}{F}_{\text {net }} \\
& =\stackrel{\rightharpoonup}{T}_{\mathrm{T}}+\stackrel{\rightharpoonup}{\mathrm{g}} \\
& =85.0 \mathrm{~N}+(-72.0 \mathrm{~N}) \\
& =13.0 \mathrm{~N} \\
\stackrel{\rightharpoonup}{F}_{\text {net }} & =m \vec{a} \\
\vec{a} & =\frac{\stackrel{\rightharpoonup}{n}_{\text {net }}}{m} \\
& =\frac{13.0 \mathrm{~N}}{36.0 \mathrm{~kg}} \\
& =0.361 \mathrm{~m} / \mathrm{s}^{2}
\end{aligned}
$$

7. Consider west as the positive direction.

$$
\begin{aligned}
\vec{v}_{\mathrm{f}}^{2} & =\vec{v}_{\mathrm{i}}^{2}+2 \vec{a} \vec{d} \\
\vec{a} & =\frac{\left(\vec{v}_{\mathrm{f}}^{2}-\vec{v}_{\mathrm{i}}^{2}\right)}{2 \vec{d}} \\
& =\frac{\left(0-(0.50 \mathrm{~m} / \mathrm{s})^{2}\right)}{2(0.25 \mathrm{~m})} \\
& =-0.50 \mathrm{~m} / \mathrm{s}^{2} \\
\vec{F}_{\mathrm{net}} & =\vec{F}_{\mathrm{f}} \\
& =m \vec{a} \\
& =(1.0 \mathrm{~kg})\left(-0.50 \mathrm{~m} / \mathrm{s}^{2}\right) \\
& =-0.50 \mathrm{~N} \\
\therefore F_{\mathrm{f}} & =0.50 \mathrm{~N}
\end{aligned}
$$

8. Consider north and up as the positive directions.


$$
\begin{aligned}
\sum \stackrel{\rightharpoonup}{F} & =\vec{F}_{\text {net }} \\
& =\vec{F}_{\text {app }}+\vec{F}_{\mathrm{f}} \\
& =2.50 \times 10^{2} \mathrm{~N}+\left(-1.40 \times 10^{2} \mathrm{~N}\right) \\
& =1.10 \times 10^{2} \mathrm{~N}
\end{aligned}
$$

$$
\begin{aligned}
\stackrel{\rightharpoonup}{F}_{\mathrm{g}} & =m \vec{g} \\
m & =\frac{\stackrel{\rightharpoonup}{F}_{g}}{\vec{g}} \\
& =\frac{-1.00 \times 10^{2} \mathrm{~N}}{-9.81 \mathrm{~m} / \mathrm{s}^{2}} \\
& =1.02 \times 10^{1} \mathrm{~kg}
\end{aligned}
$$

$$
\begin{aligned}
\stackrel{\rightharpoonup}{F}_{\text {net }} & =m \vec{a} \\
\vec{a} & =\frac{\vec{F}_{\text {net }}}{m} \\
& =\frac{1.10 \times 10^{2} \mathrm{~N}}{1.02 \times 10^{1} \mathrm{~kg}} \\
& =1.08 \times 10^{1} \mathrm{~m} / \mathrm{s}^{2} \text { or } 1.08 \times 10^{1} \mathrm{~m} / \mathrm{s}^{2} \text { north }
\end{aligned}
$$

9. Consider north as the positive direction.

$$
\begin{aligned}
\stackrel{\rightharpoonup}{F} & =m \vec{a} \\
& =(7.00 \mathrm{~kg})\left(2.30 \mathrm{~m} / \mathrm{s}^{2} \text { north }\right) \\
& =16.1 \mathrm{~N} \text { north }
\end{aligned}
$$

The magnitude of the horizontal force required to accelerate the given mass at $2.30 \mathrm{~m} / \mathrm{s}^{2}$ is 16.1 N .
10. Consider east as the positive direction. Since the car is decelerating due only to friction, the net force is actually the frictional force.

$$
\begin{aligned}
\sum \vec{F} & =\vec{F}_{\mathrm{net}}=\vec{F}_{\mathrm{f}} \\
& =m \vec{a} \\
\vec{a} & =\frac{\vec{F}_{\mathrm{f}}}{m} \\
& =\frac{-1.80 \times 10^{4} \mathrm{~N}}{1.50 \times 10^{3} \mathrm{~kg}} \\
& =-12.0 \mathrm{~m} / \mathrm{s}^{2} \\
\vec{v}_{\mathrm{f}}^{2} & =\vec{v}_{\mathrm{i}}^{2}+2 \vec{a} \vec{d} \\
0 & =(24.0 \mathrm{~m} / \mathrm{s})^{2}+2\left(-12.0 \mathrm{~m} / \mathrm{s}^{2}\right) \vec{d} \\
\vec{d} & =-\frac{(24.0 \mathrm{~m} / \mathrm{s})^{2}}{2\left(-12.0 \mathrm{~m} / \mathrm{s}^{2}\right)} \\
& =24.0 \mathrm{~m} \text { or } 24.0 \mathrm{~m} \text { east }
\end{aligned}
$$

11. Consider east to be the positive direction.

$$
\begin{aligned}
\sum \vec{F} & =\vec{F}_{\text {net }}=m \vec{a} \\
\vec{a} & =\frac{\vec{F}_{\text {net }}}{m} \\
& =\frac{-2.50 \times 10^{4} \mathrm{~N}}{1.20 \times 10^{3} \mathrm{~kg}} \\
& =-20.83 \mathrm{~m} / \mathrm{s}^{2}
\end{aligned}
$$

$$
\begin{aligned}
\vec{a} & =\frac{\vec{v}_{\mathrm{f}}-\vec{v}_{\mathrm{i}}}{t} \\
-20.83 \mathrm{~m} / \mathrm{s}^{2} & =\frac{\vec{v}_{\mathrm{f}}-20.0 \mathrm{~m} / \mathrm{s}}{0.500 \mathrm{~s}} \\
\vec{v}_{\mathrm{f}} & =\left(-20.83 \mathrm{~m} / \mathrm{s}^{2}\right)(0.500 \mathrm{~s})+20.0 \mathrm{~m} / \mathrm{s} \\
\vec{v}_{\mathrm{f}} & =9.58 \mathrm{~m} / \mathrm{s} \text { or } 9.58 \mathrm{~m} / \mathrm{s} \text { east }
\end{aligned}
$$

12. 



First, calculate the force due to gravity acting on the mass. This force is transferred through the cord to the box. This means that the force of gravity acting on the mass is also the net force acting on the entire system. The force of gravity pulls down on the mass with a force equal to its weight.
However, this force acts to accelerate both the mass and the box. Thus, the acceleration of the entire system depends on the force acting on the mass and the total mass of the mass and the box.

$$
\begin{aligned}
\sum \stackrel{\rightharpoonup}{F}_{\text {mass }} & =\stackrel{\rightharpoonup}{F}_{\text {net }}=\vec{F}_{\mathrm{T}} \\
& =m_{\text {mass }} \vec{g} \\
& =(1.5 \mathrm{~kg})\left(-9.81 \mathrm{~m} / \mathrm{s}^{2}\right) \\
& =-14.7 \mathrm{~N} \\
\sum F_{\text {sys }} & =\sum F_{\text {mass }} \\
& =m_{\text {sys }} \\
a_{\text {box }} & =\frac{\sum F_{\text {sys }}}{m_{\text {sys }}} \\
& =\frac{14.7 \mathrm{~N}}{1.0 \mathrm{~kg}+1.5 \mathrm{~kg}} \\
& =\frac{14.7 \mathrm{~N}}{2.5 \mathrm{~kg}} \\
& =5.9 \mathrm{~m} / \mathrm{s}^{2}
\end{aligned}
$$

So the box is accelerated $5.9 \mathrm{~m} / \mathrm{s}^{2}$ to the right.
13.


Consider right and up as the positive directions.

$$
\begin{aligned}
& \cos \theta=\frac{F_{\mathrm{T} x}}{F_{\mathrm{T}}} \\
& F_{\mathrm{T} x}=F_{\mathrm{T}} \cos \theta \\
&=(60.0 \mathrm{~N})\left(\cos 42.0^{\circ}\right) \\
&=44.59 \mathrm{~N} \\
& \vec{F}_{\mathrm{T} x}=44.59 \mathrm{~N} \text { or } 44.59 \mathrm{~N} \text { to the right } \\
& \vec{F}_{\mathrm{g}}= m \vec{g} \\
& m=\frac{\vec{F}_{\mathrm{g}}}{\vec{g}} \\
&=\frac{-125 \mathrm{~N}}{-9.81 \mathrm{~m} / \mathrm{s}^{2}} \\
&=12.74 \mathrm{~kg} \\
& \begin{aligned}
\sum \vec{F} & =\vec{F}_{\text {net }}=\vec{F}_{\mathrm{T} x}+\vec{F}_{\mathrm{f}} \\
& =44.59 \mathrm{~N}+(-15.0 \mathrm{~N}) \\
& =29.59 \mathrm{~N} \\
\vec{F}_{\text {net }} & =m \vec{a} \\
\vec{a} & =\frac{\vec{F}_{\mathrm{net}}}{m} \\
& =\frac{29.59 \mathrm{~N}}{12.74 \mathrm{~kg}} \\
& =2.32 \mathrm{~m} / \mathrm{s}^{2} \text { or } 2.32 \mathrm{~m} / \mathrm{s}^{2} \text { to the right }
\end{aligned}
\end{aligned}
$$

14. Consider up as the positive direction.

15. Consider north as the positive direction.

$$
\begin{aligned}
\vec{v}_{\mathrm{f}}^{2} & =\vec{v}_{\mathrm{i}}^{2}+2 \vec{a} \vec{d} \\
0 & =(3.0 \mathrm{~m} / \mathrm{s})^{2}+2(\vec{a})(8.0 \mathrm{~m}) \\
\vec{a} & =-\frac{(3.0 \mathrm{~m} / \mathrm{s})^{2}}{2(8.0 \mathrm{~m})} \\
\vec{a} & =-0.563 \mathrm{~m} / \mathrm{s}^{2} \\
\vec{F}_{\mathrm{f}} & =m \vec{a} \\
& =(0.48 \mathrm{~kg})\left(-0.563 \mathrm{~m} / \mathrm{s}^{2}\right) \\
& =-0.27 \mathrm{~N} \text { or } 0.27 \mathrm{~N} \text { south }
\end{aligned}
$$

## Practice Test

## ANSWERS AND SOLUTIONS

1. $F_{\mathrm{f}}=\mu F_{\mathrm{N}}$
$\mu=\frac{F_{\mathrm{f}}}{F_{\mathrm{N}}}$

The coefficient of friction is the ratio of the force of friction to the normal force between two surfaces in contact. The higher or lower the coefficient, the higher or lower the frictional force that exists due to the contact of the two surfaces.
2. Inertia is an object's tendency to resist a change in motion. The greater the mass of an object is, the greater the inertia.
3. The graph is linear, and this implies a direct relationship between the variables. This confirms Newton's second law, which states that the acceleration varies directly with the net force.
Mathematically, you have $a \propto F_{\text {net }}$.

$$
\begin{aligned}
\text { units of slope } & =\frac{\text { rise units }}{\text { run units }} \\
& =\frac{\mathrm{m} / \mathrm{s}^{2}}{\mathrm{~N}} \\
& =\frac{\mathrm{m} / \mathrm{s}^{2}}{\mathrm{~kg} \cdot \mathrm{~m} / \mathrm{s}^{2}} \\
& =\frac{1}{\mathrm{~kg}}=\mathrm{kg}^{-1}
\end{aligned}
$$

The slope is the reciprocal of the mass. That is, the mass of the object can be found by calculating the slope of this graph and determining its inverse.
4. a) The force of gravity and the normal force from the ground both act on the student.
b) She remains motionless because these forces are equal and opposite $\left(F_{\text {net }}=0\right)$.
5. Consider up as the positive direction.

$$
\begin{aligned}
\stackrel{\rightharpoonup}{F}_{\mathrm{g}} & =m \vec{g} \\
m & =\frac{\stackrel{\rightharpoonup}{\mathrm{F}}_{\mathrm{g}}}{\bar{g}} \\
& =\frac{-50.0 \mathrm{~N}}{-9.81 \mathrm{~m} / \mathrm{s}^{2}} \\
& =5.10 \mathrm{~kg}
\end{aligned}
$$

6. Note: You do not have a mass, and there is no way to find it. Does it matter what the mass is? Try giving the puck a mass of $m$.

| $v_{i}$ | $v_{f}$ | $a$ | $d$ | $t$ |
| :---: | :---: | :---: | :---: | :---: |
| $11 \mathrm{~m} / \mathrm{s}$ | 0 | $?$ | 25 m | $\times$ |

Consider the direction of motion as the positive direction.

$$
\begin{aligned}
\vec{v}_{\mathrm{f}}^{2} & =\vec{v}_{\mathrm{i}}^{2}+2 \vec{a} \vec{d} \\
0 & =(11 \mathrm{~m} / \mathrm{s})^{2}+2(\vec{a})(25 \mathrm{~m}) \\
\vec{a} & =-\frac{(11 \mathrm{~m} / \mathrm{s})^{2}}{2(25 \mathrm{~m})} \\
\vec{a} & =-2.42 \mathrm{~m} / \mathrm{s}^{2} \text { or } 2.42 \mathrm{~m} / \mathrm{s}^{2} \text { opposing the motion }
\end{aligned}
$$



$$
\sum \stackrel{\rightharpoonup}{F}=\stackrel{\rightharpoonup}{F}_{\mathrm{net}}=\stackrel{\rightharpoonup}{F}_{\mathrm{f}}
$$

$$
\vec{F}_{\mathrm{f}}=m \vec{a}
$$

$$
=(m)\left(-2.42 \mathrm{~m} / \mathrm{s}^{2}\right)
$$

$$
=m\left(-2.42 \mathrm{~m} / \mathrm{s}^{2}\right)
$$

$$
\begin{aligned}
\stackrel{\rightharpoonup}{\mathrm{f}} & =\mu_{\mathrm{k}} m g \\
m\left(-2.42 \mathrm{~m} / \mathrm{s}^{2}\right) & =\mu_{\mathrm{k}} m\left(-9.81 \mathrm{~m} / \mathrm{s}^{2}\right) \\
\mu_{\mathrm{k}} & =\frac{\left(-2.42 \mathrm{~m} / \mathrm{s}^{2}\right)}{\left(-9.81 \mathrm{~m} / \mathrm{s}^{2}\right)} \\
& =0.25
\end{aligned}
$$

7. The gravitational force will be toward each other. The particular direction depends on which object the force is with respect to.
8. 


$F_{\text {net }}=m \vec{a}$

$$
=(3.0 \mathrm{~kg})\left(1.2 \mathrm{~m} / \mathrm{s}^{2}\right)
$$

$$
=3.6 \mathrm{~N} \text { right }
$$

$\vec{F}_{\text {net }}=\vec{T}+\vec{F}_{\text {f }}$
$\vec{F}_{\mathrm{f}}=\vec{F}_{\text {net }}-\vec{T}$
$=3.6 \mathrm{~N}-5.2 \mathrm{~N}$
$=-1.6 \mathrm{~N}$ or 1.6 N left
9. The force of friction will remain constant at 8.0 N with a direction opposing motion until the object comes to a complete stop. Once stopped, the magnitude of the force of friction becomes zero and it has no direction.
10. a) The slope of the graph represents the acceleration. You are looking for the section that has the greatest uniform slope. A region of changing slope means that the acceleration is not constant in that region.

$$
\sum \stackrel{\rightharpoonup}{F}=\stackrel{\rightharpoonup}{F}_{n e t}=m \vec{a}
$$

The greater the acceleration (slope), the greater the net force. Section B shows the object experiencing the largest constant net force even though it is acting in the opposite direction to the motion. This is evident because the slope in section B is the most steep.
b) If the friction has the same magnitude as the applied force, the net force is zero, and the acceleration will also be zero. Section A shows an object moving at constant speed therefore zero acceleration.
11. On a horizontal surface:
$\vec{F}_{\mathrm{N}}=-\vec{F}_{\mathrm{g}}$

Because the three objects have the same masses, they all experience the same magnitude of normal force.
12. The normal force is equal and opposite to the force of gravity and the tension force is equal and opposite to the force of friction. There is no net force on the object, and therefore no acceleration.
13. a) Newton's third law applies because boy $A$ exerts force of 250 N on boy B , and boy B exerts a force of 250 N on boy A. These two forces are equal in magnitude but opposite in direction.

These two forces cannot be added in a sensible way because they act on different objects. In fact, this is the defining attribute of an actionreaction couple.
b) Boy A exerts a force of 250 N on the rope, and boy B exerts a force of 250 N on the rope in the opposite direction. The tension in the rope is 250 N throughout.

The vector sum of the forces is zero. The forces can now be added because both forces act on the rope.

$$
\begin{aligned}
\Sigma \vec{F} & =250 \mathrm{~N}+(-250 \mathrm{~N}) \\
& =0 \mathrm{~N}
\end{aligned}
$$

There would be no acceleration of either of the boys since the net force transferred to the through the rope is zero.
14.


If the object is moving at a constant velocity, there is no net force.

$$
\stackrel{\rightharpoonup}{F}_{\mathrm{g}}=-5.0 \mathrm{~N} \text { and } \stackrel{\rightharpoonup}{F}_{\mathrm{net}}=0
$$

$$
\vec{F}_{\mathrm{net}}=\vec{T}+\vec{F}_{\mathrm{g}}
$$

$$
\stackrel{\rightharpoonup}{T}=-\stackrel{\rightharpoonup}{F}_{\mathrm{g}}
$$

$$
\vec{T}=5.0 \mathrm{~N}
$$

15. Choosing right as the positive direction:

$$
\begin{aligned}
\stackrel{\rightharpoonup}{F}_{\text {net }} & =m \vec{a} \\
& =(2.0 \mathrm{~kg})\left(-1.6 \mathrm{~m} / \mathrm{s}^{2}\right) \\
& =-3.2 \mathrm{~N} \text { or } 3.2 \mathrm{~N} \text { left } \\
\vec{F}_{\text {net }} & =\vec{T}_{1}+\vec{T}_{2} \\
\vec{T}_{2} & =\vec{F}_{\text {net }}-\vec{T}_{1} \\
\vec{T}_{2} & =-3.2 \mathrm{~N}-(-17 \mathrm{~N}) \\
\vec{T}_{2} & =14 \mathrm{~N} \text { right } \\
T_{2} & =14 \mathrm{~N}
\end{aligned}
$$

16. $\vec{F}_{\text {net }}=0$
$\vec{F}_{\mathrm{g}}=m \bar{g}$

$$
=(2.00 \mathrm{~kg})\left(-9.81 \mathrm{~m} / \mathrm{s}^{2}\right)
$$

$$
=-19.62 \mathrm{~N} \text { or } 19.62 \mathrm{~N} \text { down }
$$

$\vec{F}_{\text {net }}=\vec{F}_{\mathrm{s}}+\vec{F}_{\mathrm{g}}$
$\vec{F}_{\mathrm{s}}=\vec{F}_{\text {net }}-\vec{F}_{\mathrm{g}}$

$$
=0-(-19.62 \mathrm{~N})
$$

$\vec{F}_{\mathrm{s}}=19.62 \mathrm{~N}$ up
$\vec{F}_{\mathrm{s}}=-k \vec{x}$
$k=-\frac{\vec{F}_{\mathrm{s}}}{\vec{x}}$
$k=-\frac{19.62 \mathrm{~N}}{-9.00 \times 10^{-2} \mathrm{~m}}$
$k=218 \mathrm{~N} / \mathrm{m}$
17. The net force can be determined from the free-body diagram. The tension in the cord is transferred around the pulley and acts on the mass in the upward direction. Consider up as the positive direction.


Now, the acceleration of the system can be determined, and from that, the time can be calculated.

$$
\begin{aligned}
\vec{a} & =\frac{\stackrel{\rightharpoonup}{F}_{\text {net }}}{m} \\
& =\frac{35.95 \mathrm{~N}}{5.0 \mathrm{~kg}} \\
& =7.19 \mathrm{~m} / \mathrm{s}^{2} \text { or } 7.19 \mathrm{~m} / \mathrm{s}^{2}
\end{aligned}
$$

$\vec{d}=\vec{v}_{\mathrm{i}} t+\frac{1}{2} \vec{a} t^{2}$
$\vec{d}=0+\frac{1}{2} \vec{a} t^{2}$
$t=\sqrt{\frac{2 \vec{d}}{\vec{a}}}$
$t=\sqrt{\frac{2(1.00 \mathrm{~m})}{7.19 \mathrm{~m} / \mathrm{s}^{2}}}$
$=0.527 \mathrm{~s}$
18. a) Consider up and right as the positive directions.

$$
\begin{aligned}
\vec{F}_{\mathrm{N}} & =-\vec{F}_{\mathrm{g}} \\
& =-m \vec{g} \\
& =-(7.0 \mathrm{~kg})\left(-9.81 \mathrm{~m} / \mathrm{s}^{2}\right) \\
& =68.6 \mathrm{~N} \text { or } 68.6 \mathrm{~N} \mathrm{up}
\end{aligned}
$$

$$
\begin{aligned}
F_{\mathrm{f}} & =\mu_{\mathrm{k}} F_{\mathrm{N}} \\
\mu_{\mathrm{k}} & =\frac{F_{\mathrm{f}}}{F_{\mathrm{N}}} \\
& =\frac{25 \mathrm{~N}}{68.6 \mathrm{~N}} \\
& =0.36
\end{aligned}
$$

b) Consider up and right as the positive directions.
The force of friction between the smaller mass and the underlying surface must be determined.

$$
\begin{aligned}
\vec{F}_{\mathrm{N}} & =-\vec{F}_{\mathrm{g}} \\
\vec{F}_{\mathrm{N}} & =-m \vec{g} \\
& =-(2.5 \mathrm{~kg})\left(-9.81 \mathrm{~m} / \mathrm{s}^{2}\right) \\
& =24.5 \mathrm{~N} \\
\vec{F}_{\mathrm{f}} & =\mu \vec{F}_{\mathrm{N}} \\
\vec{F}_{\mathrm{f}} & =(0.63)(24.5 \mathrm{~N}) \\
& =15.5 \mathrm{~N}
\end{aligned}
$$

The total force of kinetic friction along with the net force can be used to determine the applied force.

$$
\begin{aligned}
\Sigma \stackrel{\rightharpoonup}{F} & =\stackrel{\rightharpoonup}{F}+\stackrel{\rightharpoonup}{f} \\
m \vec{a} & =\vec{F}_{\mathrm{A}}+\bar{F}_{\mathrm{f}} \\
\vec{F}_{\mathrm{A}} & =m \vec{a}-\vec{F}_{\mathrm{f}} \\
\vec{F}_{\mathrm{A}} & =(9.5 \mathrm{~kg})\left(2.5 \mathrm{~m} / \mathrm{s}^{2}\right)-(-25 \mathrm{~N}+(-15.4 \mathrm{~N})) \\
\vec{F}_{\mathrm{A}} & =64 \mathrm{~N} \text { or } 64 \mathrm{~N} \text { to the right }
\end{aligned}
$$

19. 



$$
\begin{aligned}
\stackrel{\rightharpoonup}{F}_{\text {net }} & =m \vec{a} \\
& =(1.50 \mathrm{~kg})\left(1.20 \mathrm{~m} / \mathrm{s}^{2}\right) \\
& =1.80 \mathrm{~N} \text { up } \\
\stackrel{\rightharpoonup}{F}_{\mathrm{g}} & =m \vec{g} \\
& =(1.50 \mathrm{~kg})\left(-9.80 \mathrm{~m} / \mathrm{s}^{2}\right) \\
& =-14.7 \mathrm{~N} \text { or } 14.7 \mathrm{~N} \text { down } \\
\stackrel{\rightharpoonup}{F}_{\text {net }} & =\vec{F}_{\mathrm{a}}+\vec{F}_{\mathrm{g}} \\
\stackrel{\rightharpoonup}{F}_{\mathrm{a}} & =\vec{F}_{\text {net }}-\vec{F}_{\mathrm{g}} \\
& =1.80 \mathrm{~N}-(-14.7 \mathrm{~N}) \\
& =16.5 \mathrm{~N} \text { up }
\end{aligned}
$$

20. a) The free-body diagram shows that the applied force resolved into horizontal and vertical components.


The upward component of the force plus the normal force is equal and opposite the gravitational force.

$$
\begin{aligned}
\stackrel{\rightharpoonup}{\mathrm{g}}_{\mathrm{g}} & =-\left(\stackrel{\rightharpoonup}{F}_{\mathrm{N}}+F \sin \theta\right) \\
& =-\vec{F}_{\mathrm{N}}-F \sin \theta \\
\stackrel{\rightharpoonup}{F}_{\mathrm{N}} & =-\vec{F}_{\mathrm{g}}-F \sin \theta \\
& =-m \bar{g}-F \sin \theta \\
& =-(10.0 \mathrm{~kg})\left(-9.81 \mathrm{~m} / \mathrm{s}^{2}\right)-(100 \mathrm{~N})\left(\sin 30.0^{\circ}\right) \\
& =48.1 \mathrm{~N} \text { or } 48.1 \mathrm{~N} \text { up }
\end{aligned}
$$

b) Use the calculation from the previous problem to calculate the magnitude of the force of friction.

$$
\begin{aligned}
\stackrel{\rightharpoonup}{F}_{\mathrm{g}} & =-\left(\vec{F}_{\mathrm{N}}+F \sin \theta\right) \\
& =-\vec{F}_{\mathrm{N}}-F \sin \theta \\
\stackrel{\rightharpoonup}{F}_{\mathrm{N}} & =-\vec{F}_{\mathrm{g}}-F \sin \theta \\
& =-m \vec{g}-F \sin \theta \\
& =-(10.0 \mathrm{~kg})\left(-9.81 \mathrm{~m} / \mathrm{s}^{2}\right)-(100 \mathrm{~N})\left(\sin 30.0^{\circ}\right) \\
& =48.1 \mathrm{~N} \text { or } 48.1 \mathrm{Nup} \\
F_{\mathrm{f}} & =\mu_{\mathrm{k}} F_{\mathrm{N}} \\
F_{\mathrm{f}} & =(0.620)(48.1 \mathrm{~N}) \\
& =29.8 \mathrm{~N}
\end{aligned}
$$

c) Consider right as the positive direction.

$$
\begin{aligned}
\vec{F}_{\text {net }} & =F \cos \theta+\vec{F}_{\mathrm{f}} \\
& =(100 \mathrm{~N}) \cos 30.0^{\circ}+(-29.8 \mathrm{~N}) \\
& =56.8 \mathrm{~N} \\
\vec{F}_{\text {net }} & =m \vec{a} \\
\vec{a} & =\frac{\vec{F}_{\text {net }}}{m} \\
& =\frac{56.8 \mathrm{~N}}{10.0 \mathrm{~kg}} \\
& =5.68 \mathrm{~m} / \mathrm{s}^{2}
\end{aligned}
$$

## ENERGY

## Lesson 1—Work

## PRACTICE EXERCISES

ANSWERS AND SOLUTIONS

1. $W=F d$

$$
\begin{aligned}
& =(20.0 \mathrm{~N})(1.50 \mathrm{~m}) \\
& =30.0 \mathrm{~J}
\end{aligned}
$$

2. $W=F d$

$$
\begin{aligned}
& =(6.00 \mathrm{~N})(3.00 \mathrm{~m}) \\
& =18.0 \mathrm{~J}
\end{aligned}
$$

3. $W=F d$

$$
\begin{aligned}
& =(2.20 \mathrm{~N})(0) \\
& =0
\end{aligned}
$$

4. $\vec{d}=\left(\frac{\stackrel{\rightharpoonup}{\mathrm{v}}_{\mathrm{f}}+\stackrel{\rightharpoonup}{v}_{0}}{2}\right) t$

$$
\begin{aligned}
& =\left(\frac{11.0 \mathrm{~m} / \mathrm{s}+0}{2}\right)(5.00 \mathrm{~s}) \\
& =27.5 \mathrm{~m} \\
\vec{a} & =\frac{\vec{v}_{\mathrm{f}}-\vec{v}_{0}}{t} \\
& =\frac{11.0 \mathrm{~m} / \mathrm{s}-0}{5.00 \mathrm{~s}} \\
& =+2.20 \mathrm{~m} / \mathrm{s}^{2}
\end{aligned}
$$

$$
\vec{F}=m \vec{a}
$$

$$
=(10.0 \mathrm{~kg})\left(+2.20 \mathrm{~m} / \mathrm{s}^{2}\right)
$$

$$
=+22.0 \mathrm{~N}
$$

$$
W=F d
$$

$$
=(22.0 \mathrm{~N})(27.5 \mathrm{~m})
$$

$$
=605 \mathrm{~J}
$$

5. Find the horizontal component of the force.


$$
\begin{aligned}
& \cos \theta=\frac{\text { adjacent }}{\text { hypotenuse }} \\
& \text { adjacent }=\left(\cos 20.0^{\circ}\right)(75.0 \mathrm{~N}) \\
&=70.48 \mathrm{~N} \\
& \begin{aligned}
W & =F d \\
& =(70.48 \mathrm{~N})(10.0 \mathrm{~m}) \\
& =705 \mathrm{~J}
\end{aligned}
\end{aligned}
$$

6. $W=m g h$

$$
\begin{aligned}
& =(60.0 \mathrm{~kg})\left(9.81 \mathrm{~m} / \mathrm{s}^{2}\right)(3.2 \mathrm{~m}) \\
& =1.9 \times 10^{3} \mathrm{~J}
\end{aligned}
$$

7. $W=F d$

$$
\begin{aligned}
& =(0)(9.0 \mathrm{~m}) \\
& =0
\end{aligned}
$$

8. $W=m g h$

$$
\begin{aligned}
& =(80.0 \mathrm{~kg})\left(9.81 \mathrm{~m} / \mathrm{s}^{2}\right)(7.0 \mathrm{~m}) \\
& =5.5 \times 10^{3} \mathrm{~J}
\end{aligned}
$$

9. Work done against friction:

$$
\begin{aligned}
W & =F d \\
& =(3.8 \mathrm{~N})(6.0 \mathrm{~m}) \\
& =22.8 \mathrm{~J}
\end{aligned}
$$

Work done to accelerate object:

$$
\begin{aligned}
\vec{d} & =\vec{v}_{0} t+\frac{1}{2} \vec{a} t^{2} \\
6.0 \mathrm{~m} & =\frac{1}{2} \vec{a}(4.0 \mathrm{~s})^{2} \\
\vec{a} & =0.750 \mathrm{~m} / \mathrm{s}^{2}
\end{aligned}
$$

$$
\begin{aligned}
\vec{F} & =m \vec{a} \\
& =(25.0 \mathrm{~kg})\left(0.750 \mathrm{~m} / \mathrm{s}^{2}\right) \\
& =18.8 \mathrm{~N}
\end{aligned}
$$

$$
\begin{aligned}
W & =F d \\
& =(18.8 \mathrm{~N})(6.0 \mathrm{~m}) \\
& =1.13 \times 10^{2} \mathrm{~J}
\end{aligned}
$$

The total work done is:

$$
\begin{aligned}
W & =1.13 \times 10^{2} \mathrm{~J}+22.8 \mathrm{~J} \\
& =1.4 \times 10^{2} \mathrm{~J}
\end{aligned}
$$

10. Convert $55 \mathrm{~km} / \mathrm{h}$ to $\mathrm{m} / \mathrm{s}$ :

$$
\begin{aligned}
& 55 \mathrm{~km} / \mathrm{h} \times 1000 \mathrm{~m} / \mathrm{km} \times \frac{1 \mathrm{~h}}{3600 \mathrm{~s}}=15.3 \mathrm{~m} / \mathrm{s} \\
& \vec{v}_{\mathrm{f}}^{2}=\vec{v}_{0}^{2}+2 \vec{a} \vec{d} \\
& 0=(15.3 \mathrm{~m} / \mathrm{s})^{2}+2 \vec{a}(38 \mathrm{~m}) \\
& \vec{a}=-3.08 \mathrm{~m} / \mathrm{s} \\
& \vec{F}=m \vec{a} \\
&=(1165 \mathrm{~kg})\left(-3.08 \mathrm{~m} / \mathrm{s}^{2}\right) \\
&=-3.59 \times 10^{3} \mathrm{~N} \\
& W=F d \\
&=\left(-3.59 \times 10^{3} \mathrm{~N}\right)(38 \mathrm{~m}) \\
&=-1.4 \times 10^{5} \mathrm{~J}
\end{aligned}
$$

11. Work is the area under a force-displacement graph.
$W=F d$

$$
\begin{aligned}
& =(3.5 \mathrm{~N})(16.0 \mathrm{~m}) \\
& =56 \mathrm{~J}
\end{aligned}
$$

12. Work is the area under the force-displacement graph.

$$
\begin{aligned}
\text { area } & =\frac{1}{2}(l \times w)+(l \times w) \\
& =\frac{1}{2}(3.0 \mathrm{~m} \times(-4.0 \mathrm{~N}))+(5.0 \mathrm{~m} \times(-4.0 \mathrm{~N})) \\
& =-26 \mathrm{~J}
\end{aligned}
$$

## Lesson 2-Energy

## PRACTICE EXERCISES ANSWERS AND SOLUTIONS

1. $E_{\mathrm{p}}=F d$

$$
\begin{aligned}
& =(25.0 \mathrm{~N})(2.10 \mathrm{~m}) \\
& =52.5 \mathrm{~J}
\end{aligned}
$$

2. $E_{\mathrm{p}}=F d$

$$
\begin{aligned}
& =(65.0 \mathrm{~N})\left(6.5 \times 10^{-2} \mathrm{~m}\right) \\
& =4.2 \mathrm{~J}
\end{aligned}
$$

3. $E_{\mathrm{p}}=m g h$

$$
\begin{aligned}
& =(2.75 \mathrm{~kg})\left(9.81 \mathrm{~m} / \mathrm{s}^{2}\right)(7.00 \mathrm{~m}) \\
& =189 \mathrm{~J}
\end{aligned}
$$

4. $E_{\mathrm{p}}=m g h$

$$
\begin{aligned}
& =(2.0 \mathrm{~kg})\left(9.81 \mathrm{~m} / \mathrm{s}^{2}\right)(0.25 \mathrm{~m}) \\
& =4.9 \mathrm{~J}
\end{aligned}
$$

5. $E_{\mathrm{p}}=m g h$

$$
\begin{aligned}
& =\left(2.00 \times 10^{2} \mathrm{~kg}\right)\left(9.81 \mathrm{~m} / \mathrm{s}^{2}\right)(6.0 \mathrm{~m}) \\
& =1.2 \times 10^{4} \mathrm{~J}
\end{aligned}
$$

## Lesson 3-Kinetic Energy

## PRACTICE EXERCISES

 ANSWERS AND SOLUTIONS1. $E_{\mathrm{k}}=\frac{1}{2} m v^{2}$

$$
\begin{aligned}
& =\frac{1}{2}(3.0 \mathrm{~kg})(7.5 \mathrm{~m} / \mathrm{s})^{2} \\
& =84 \mathrm{~J}
\end{aligned}
$$

2. $\bar{F}_{\mathrm{g}}=m \bar{g}$

$$
\begin{aligned}
m & =\frac{\bar{F}_{g}}{\bar{g}} \\
& =\frac{-20.0 \mathrm{~N}}{-9.81 \mathrm{~m} / \mathrm{s}^{2}} \\
& =2.039 \mathrm{~kg}
\end{aligned}
$$

$$
E_{\mathrm{k}}=\frac{1}{2} m v^{2}
$$

$$
v=\sqrt{\frac{2 E_{\mathrm{k}}}{m}}
$$

$$
\begin{aligned}
& =\sqrt{\frac{2\left(5.00 \times 10^{2} \mathrm{~J}\right)}{2.039 \mathrm{~kg}}} \\
& =22.1 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

3. $\bar{v}_{\mathrm{f}}^{2}=\bar{v}_{0}^{2}+2 \bar{a} \vec{d}$

$$
\begin{aligned}
& =2\left(2.5 \mathrm{~m} / \mathrm{s}^{2}\right)(15.0 \mathrm{~m}) \\
\stackrel{\rightharpoonup}{\mathrm{v}}_{\mathrm{f}} & =8.66 \mathrm{~m} / \mathrm{s} \\
\stackrel{\rightharpoonup}{\mathrm{~F}}_{\mathrm{g}} & =m \bar{g} \\
m & =\frac{\bar{F}_{\mathrm{g}}}{\bar{g}} \\
& =\frac{-10.0 \mathrm{~N}}{-9.81 \mathrm{~m} / \mathrm{s}^{2}} \\
& =1.02 \mathrm{~kg}
\end{aligned}
$$

$$
\begin{aligned}
E_{\mathrm{k}} & =\frac{1}{2} m v^{2} \\
& =\frac{1}{2}(1.02 \mathrm{~kg})(8.66 \mathrm{~m} / \mathrm{s})^{2} \\
& =38 \mathrm{~J}
\end{aligned}
$$

4. $\vec{v}_{\mathrm{f}}^{2}=\bar{v}_{0}^{2}+2 \vec{a} \vec{d}$

$$
\begin{aligned}
& =2\left(-9.81 \mathrm{~m} / \mathrm{s}^{2}\right)(-7.0 \mathrm{~m}) \\
{\overrightarrow{{ }_{\mathrm{v}}^{\mathrm{f}}}} & =-11.7 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

$$
\begin{aligned}
E_{\mathrm{k}} & =\frac{1}{2} m v^{2} \\
& =\frac{1}{2}(8.0 \mathrm{~kg})(11.7 \mathrm{~m} / \mathrm{s})^{2} \\
& =5.5 \times 10^{2} \mathrm{~J}
\end{aligned}
$$

5. $\quad \vec{F}_{\mathrm{g}}=m \bar{g}$

$$
\begin{aligned}
m & =\frac{\vec{F}_{\mathrm{g}}}{\bar{g}} \\
& =\frac{-10.0 \mathrm{~N}}{-9.8!\mathrm{m} / \mathrm{s}^{2}} \\
& =1.02 \mathrm{~kg}
\end{aligned}
$$

$$
\begin{aligned}
E_{\mathrm{k}} & =\frac{1}{2} m v^{2} \\
v & =\sqrt{\frac{2 E_{\mathrm{k}}}{m}} \\
& =\sqrt{\frac{2\left(3.00 \times 10^{2} \mathrm{~J}\right)}{1.02 \mathrm{~kg}}} \\
& =24.3 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

## Lesson 4-Law of Conservation of Energy

## PRACTICE EXERCISES

 ANSWERS AND SOLUTIONS1. $\Delta E_{\mathrm{k}}+\Delta E_{\mathrm{p}}=0$

$$
\begin{aligned}
& \Delta E_{\mathrm{k}}=-\Delta E_{\mathrm{p}} \\
& \frac{1}{2} m\left(v_{\mathrm{f}}^{2}-v_{0}^{2}\right)=-m g \Delta h \\
& \frac{1}{2}\left(v_{\mathrm{f}}^{2}-v_{0}^{2}\right)=-g\left(h_{\mathrm{f}}-h_{0}\right) \\
& \frac{1}{2}\left((3.2 \mathrm{~m} / \mathrm{s})^{2}-0\right)=-\left(9.81 \mathrm{~m} / \mathrm{s}^{2}\right)\left(0-h_{0}\right) \\
& h_{0}=\frac{\frac{1}{2}(3.2 \mathrm{~m} / \mathrm{s})^{2}}{\left(9.81 \mathrm{~m} / \mathrm{s}^{2}\right)} \\
&=0.52 \mathrm{~m}
\end{aligned}
$$

2. $\Delta E_{\mathrm{k}}+\Delta E_{\mathrm{p}}=0$

$$
\begin{aligned}
\Delta E_{\mathrm{k}} & =-\Delta E_{\mathrm{p}} \\
\frac{1}{2} m\left(v_{\mathrm{f}}^{2}-v_{0}^{2}\right) & =-m g \Delta h \\
\frac{1}{2}\left(v_{\mathrm{f}}^{2}-v_{0}^{2}\right) & =-g \Delta h \\
\frac{1}{2}\left(v_{\mathrm{f}}^{2}-0\right) & =-\left(9.81 \mathrm{~m} / \mathrm{s}^{2}\right)(-8.0 \mathrm{~m}) \\
v_{\mathrm{f}} & =\sqrt{-\left(9.81 \mathrm{~m} / \mathrm{s}^{2}\right)(-8.0 \mathrm{~m})(2)} \\
& =13 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

3. $\Delta E_{\mathrm{k}}+\Delta E_{\mathrm{p}}=0$

$$
\begin{aligned}
& \Delta E_{\mathrm{k}}=-\Delta E_{\mathrm{p}} \\
& \frac{1}{2} m\left(v_{\mathrm{f}}^{2}-v_{0}^{2}\right)=-m g \Delta h \\
& \frac{1}{2}\left(v_{\mathrm{f}}^{2}-v_{0}^{2}\right)=-g\left(h_{\mathrm{f}}-h_{0}\right) \\
& \frac{1}{2}\left((37.0 \mathrm{~m} / \mathrm{s})^{2}-0\right)=-\left(9.81 \mathrm{~m} / \mathrm{s}^{2}\right)\left(0-h_{0}\right) \\
& h_{0}=\frac{\frac{1}{2}(37.0 \mathrm{~m} / \mathrm{s})^{2}}{9.81 \mathrm{~m} / \mathrm{s}^{2}} \\
&=69.8 \mathrm{~m}
\end{aligned}
$$

4. 

$$
\begin{aligned}
& \Delta E_{\mathrm{k}}+\Delta E_{\mathrm{p}}=0 \\
& \Delta E_{\mathrm{k}}=-\Delta E_{\mathrm{p}} \\
& \frac{1}{2} m\left(v_{\mathrm{f}}^{2}-v_{0}^{2}\right)=-m g \Delta h \\
& \frac{1}{2}\left(v_{\mathrm{f}}^{2}-v_{0}^{2}\right)=-g \Delta h \\
& \frac{1}{2}\left(v_{\mathrm{f}}^{2}-(11.0 \mathrm{~m} / \mathrm{s})^{2}\right)=-\left(9.81 \mathrm{~m} / \mathrm{s}^{2}\right)\left(-1.3 \times 10^{2} \mathrm{~m}\right) \\
& v_{\mathrm{f}}=\sqrt{-\left(9.81 \mathrm{~m} / \mathrm{s}^{2}\right)\left(-1.3 \times 10^{2} \mathrm{~m}\right)(2)+(11.0 \mathrm{~m} / \mathrm{s})^{2}} \\
&=52 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

5. $\Delta E_{\mathrm{k}}+\Delta E_{\mathrm{p}}=0$
$\Delta E_{\mathrm{k}}=-\Delta E_{\mathrm{p}}$
$\frac{1}{2} m\left(v_{\mathrm{f}}^{2}-v_{0}^{2}\right)=-m g \Delta h$
$\frac{1}{2}\left(v_{f}^{2}-v_{0}^{2}\right)=-g \Delta h$

$$
\begin{aligned}
\frac{1}{2}\left(v_{\mathrm{f}}^{2}-0\right) & =-\left(9.81 \mathrm{~m} / \mathrm{s}^{2}\right)(-4.0 \mathrm{~m}) \\
v_{\mathrm{f}} & =\sqrt{-\left(9.81 \mathrm{~m} / \mathrm{s}^{2}\right)(-4.0 \mathrm{~m})(2)} \\
& =8.9 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

6. $\Delta E_{\mathrm{k}}+\Delta E_{\mathrm{p}}=0$

$$
\begin{aligned}
\Delta E_{\mathrm{k}} & =-\Delta E_{\mathrm{p}} \\
\frac{1}{2} m\left(v_{\mathrm{f}}^{2}-v_{0}^{2}\right) & =-m g \Delta h \\
\frac{1}{2}\left(v_{\mathrm{f}}^{2}-v_{0}^{2}\right) & =-g \Delta h \\
\frac{1}{2}\left(v_{\mathrm{f}}^{2}-0\right) & =-\left(9.81 \mathrm{~m} / \mathrm{s}^{2}\right)(-0.25 \mathrm{~m}) \\
v_{\mathrm{f}} & =\sqrt{-\left(9.81 \mathrm{~m} / \mathrm{s}^{2}\right)(-0.25 \mathrm{~m})(2)} \\
& =2.2 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

7. Find $h 0$ first:

$$
\begin{aligned}
\sin 30.0^{\circ} & =\frac{h_{0}}{12.0 \mathrm{~m}} \\
h_{0} & =6.0 \mathrm{~m} \\
\Delta E_{\mathrm{k}}+\Delta E_{\mathrm{p}} & =0 \\
\Delta E_{\mathrm{k}} & =-\Delta E_{\mathrm{p}} \\
\frac{1}{2} m\left(v_{\mathrm{f}}^{2}-v_{0}^{2}\right) & =-m g \Delta h \\
\frac{1}{2}\left(v_{\mathrm{f}}^{2}-v_{0}^{2}\right) & =-g \Delta h \\
\frac{1}{2}\left(v_{\mathrm{f}}^{2}-0\right) & =-\left(9.81 \mathrm{~m} / \mathrm{s}^{2}\right)(-6.0 \mathrm{~m}) \\
v_{\mathrm{f}} & =\sqrt{-\left(9.81 \mathrm{~m} / \mathrm{s}^{2}\right)(-6.0 \mathrm{~m})(2)} \\
& =10.8 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

8. $\Delta E_{\mathrm{k}}+\Delta E_{\mathrm{p}}=0$

$$
\begin{aligned}
\Delta E_{\mathrm{k}} & =-\Delta E_{\mathrm{p}} \\
\frac{1}{2} m\left(v_{\mathrm{f}}^{2}-v_{0}^{2}\right) & =-m g \Delta h \\
\frac{1}{2}\left(v_{\mathrm{f}}^{2}-v_{0}^{2}\right) & =-g \Delta h \\
\frac{1}{2}\left(v_{\mathrm{f}}^{2}-0\right) & =-\left(9.81 \mathrm{~m} / \mathrm{s}^{2}\right)(-8.0 \mathrm{~m}) \\
v_{\mathrm{f}} & =\sqrt{-\left(9.81 \mathrm{~m} / \mathrm{s}^{2}\right)(-8.0 \mathrm{~m})(2)} \\
& =13 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

9. $\Delta E_{\mathrm{k}}+\Delta E_{\mathrm{p}}=0$

$$
\begin{aligned}
\Delta E_{\mathrm{k}} & =-\Delta E_{\mathrm{p}} \\
\frac{1}{2} m\left(v_{\mathrm{f}}^{2}-v_{0}^{2}\right) & =-m g \Delta h \\
\frac{1}{2}\left(v_{\mathrm{f}}^{2}-v_{0}^{2}\right) & =-g \Delta h \\
\frac{1}{2}\left(v_{\mathrm{f}}^{2}-0\right) & =-\left(9.81 \mathrm{~m} / \mathrm{s}^{2}\right)(-10.0 \mathrm{~m}) \\
v_{\mathrm{f}} & =\sqrt{-\left(9.81 \mathrm{~m} / \mathrm{s}^{2}\right)(-10.0 \mathrm{~m})(2)} \\
& =14 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

10. $\Delta E_{\mathrm{k}}+\Delta E_{\mathrm{p}}=0$

$$
\Delta E_{\mathrm{k}}=-\Delta E_{\mathrm{p}}
$$

$$
\frac{1}{2} m\left(v_{\mathrm{f}}^{2}-v_{0}^{2}\right)=-m g \Delta h
$$

$$
\frac{1}{2}\left(v_{\mathrm{f}}^{2}-v_{0}^{2}\right)=-g \Delta h
$$

$$
\begin{aligned}
\Delta h & =\frac{\frac{1}{2}\left(0-(3.5 \mathrm{~m} / \mathrm{s})^{2}\right)}{-\left(9.81 \mathrm{~m} / \mathrm{s}^{2}\right)} \\
& =0.62 \mathrm{~m}
\end{aligned}
$$

11. 



Find $\Delta h$ first:

$$
\begin{aligned}
& \cos 25^{\circ}=\frac{y}{1.2 \mathrm{~m}} \\
& y=1.09 \mathrm{~m} \\
& \therefore \Delta h=1.2 \mathrm{~m}-y \\
&= .2 \mathrm{~m}-1.09 \mathrm{~m} \\
&=0.11 \mathrm{~m} \\
& \Delta E_{\mathrm{k}}+\Delta E_{\mathrm{p}}=0 \\
& \Delta E_{\mathrm{k}}=-\Delta E_{\mathrm{p}} \\
& \frac{1}{2} m\left(v_{\mathrm{f}}^{2}-v_{0}^{2}\right)=-m g \Delta h \\
& \frac{1}{2}\left(v_{\mathrm{f}}^{2}-v_{0}^{2}\right)=-g \Delta h \\
& \frac{1}{2}\left(v_{\mathrm{f}}^{2}-0\right)=-\left(9.81 \mathrm{~m} / \mathrm{s}^{2}\right)(-0.11 \mathrm{~m}) \\
& v_{\mathrm{f}}=\sqrt{-\left(9.81 \mathrm{~m} / \mathrm{s}^{2}\right)(-0.11 \mathrm{~m})(2)} \\
&=1.47 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

12. Note: The initial vertical speed is zero.

$$
\begin{aligned}
\Delta E_{\mathrm{k}}+\Delta E_{\mathrm{p}} & =0 \\
\Delta E_{\mathrm{k}} & =-\Delta E_{\mathrm{p}} \\
\frac{1}{2} m\left(v_{\mathrm{f}}^{2}-v_{0}^{2}\right) & =-m g \Delta h \\
\frac{1}{2}\left(v_{\mathrm{f}}^{2}-v_{0}^{2}\right) & =-g \Delta h \\
\frac{1}{2}\left(v_{\mathrm{f}}^{2}-0\right) & =-\left(9.81 \mathrm{~m} / \mathrm{s}^{2}\right)(-5.0 \mathrm{~m}) \\
v_{\mathrm{f}} & =\sqrt{-\left(9.81 \mathrm{~m} / \mathrm{s}^{2}\right)(-5.0 \mathrm{~m})(2)} \\
& =9.9 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

13. $\Delta E_{\mathrm{k}}+\Delta E_{\mathrm{p}}=0$

$$
\begin{aligned}
\Delta E_{\mathrm{k}} & =-\Delta E_{\mathrm{p}} \\
\frac{1}{2} m\left(v_{\mathrm{f}}^{2}-v_{0}^{2}\right) & =-m g \Delta h \\
\frac{1}{2}\left(v_{\mathrm{f}}^{2}-v_{0}^{2}\right) & =-g \Delta h \\
\frac{1}{2}\left(v_{\mathrm{f}}^{2}-0\right) & =-\left(9.81 \mathrm{~m} / \mathrm{s}^{2}\right)(-2.0 \mathrm{~m}) \\
v_{\mathrm{f}} & =\sqrt{-\left(9.81 \mathrm{~m} / \mathrm{s}^{2}\right)(-2.0 \mathrm{~m})(2)} \\
& =6.3 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

## Lesson 5-Power

PRACTICE EXERCISES ANSWERS AND SOLUTIONS

1. $\quad P=\frac{W}{t}$ and $W=m g h$

$$
t=\frac{m g h}{P}
$$

$$
\begin{aligned}
& =\frac{(45.0 \mathrm{~kg})\left(9.81 \mathrm{~m} / \mathrm{s}^{2}\right)(6.0 \mathrm{~m})}{1.50 \times 10^{3} \mathrm{~W}} \\
& =1.8 \mathrm{~s}
\end{aligned}
$$

2. $\quad P=\frac{W}{t}$ and $W=m g h$

$$
\begin{aligned}
P & =\frac{m g h}{t} \\
P & =\frac{(20.0 \mathrm{~kg})\left(9.81 \mathrm{~m} / \mathrm{s}^{2}\right)(2.50 \mathrm{~m})}{2.00 \mathrm{~s}} \\
& =245 \mathrm{~W}
\end{aligned}
$$

3. $\vec{v}_{\mathrm{av}}=\frac{\stackrel{\rightharpoonup}{\mathrm{f}}_{\mathrm{f}}+\vec{v}_{0}}{2}$

$$
\begin{aligned}
& =\frac{3.00 \mathrm{~m} / \mathrm{s}+0}{2} \\
& =1.50 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

$$
\vec{v}_{\mathrm{f}}^{2}=\vec{v}_{0}^{2}+2 \vec{a} \vec{d}
$$

$$
(3.00 \mathrm{~m} / \mathrm{s})^{2}=2 \bar{a}(1.5 \mathrm{~m})
$$

$$
\vec{a}=3.0 \mathrm{~m} / \mathrm{s}^{2}
$$

$\vec{F}=m \vec{a}$

$$
\begin{aligned}
& =(2.00 \mathrm{~kg})\left(3.0 \mathrm{~m} / \mathrm{s}^{2}\right) \\
& =6.0 \mathrm{~N}
\end{aligned}
$$

$$
\begin{aligned}
P & =F v_{\mathrm{av}} \\
& =(6.0 \mathrm{~N})(1.50 \mathrm{~m} / \mathrm{s}) \\
& =9.0 \mathrm{~W}
\end{aligned}
$$

4. $\quad$ power out $=F v$ and $F_{\mathrm{a}}=m g$

$$
\begin{aligned}
\text { power out } & =m g v \\
& =\left(8.5 \times 10^{2} \mathrm{~kg}\right)\left(9.80 \mathrm{~m} / \mathrm{s}^{2}\right)(1.00 \mathrm{~m} / \mathrm{s}) \\
& =8.33 \times 10^{3} \mathrm{~W}
\end{aligned}
$$

$$
\text { efficiency }=\frac{\text { power out }}{\text { power in }} \times 100 \%
$$

$$
=\frac{8.33 \times 10^{3} \mathrm{~W}}{10.0 \times 10^{3} \mathrm{~W}} \times 100 \%
$$

$$
=83.3 \%
$$

5. $\quad \vec{v}_{\mathrm{f}}^{2}=\vec{v}_{0}^{2}+2 \vec{a} \vec{d}$

$$
(6.0 \mathrm{~m} / \mathrm{s})^{2}=2 \vec{a}(2.0 \mathrm{~m})
$$

$$
\bar{a}=+9.00 \mathrm{~m} / \mathrm{s}^{2}
$$

$$
\begin{aligned}
\vec{F}_{\mathrm{net}} & =m \vec{a} \\
& =(5.0 \mathrm{~kg})\left(9.00 \mathrm{~m} / \mathrm{s}^{2}\right) \\
& =+45.0 \mathrm{~N}
\end{aligned}
$$

$$
\begin{aligned}
\stackrel{\rightharpoonup}{F}_{\mathrm{net}} & =\vec{F}_{\mathrm{a}}+\stackrel{\rightharpoonup}{\mathrm{f}} \\
\stackrel{\rightharpoonup}{F}_{\mathrm{a}} & =\vec{F}_{\text {net }}-\stackrel{\rightharpoonup}{F}_{\mathrm{f}} \\
& =45.0 \mathrm{~N}-(-4.0 \mathrm{~N}) \\
& =+49.0 \mathrm{~N}
\end{aligned}
$$

$$
\vec{v}_{\mathrm{av}}=\frac{\vec{v}_{\mathrm{f}}+\vec{v}_{0}}{2}
$$

$$
\begin{aligned}
& =\frac{6.0 \mathrm{~m} / \mathrm{s}+0}{2} \\
& =+3.00 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

$$
P=F_{\mathrm{a}} v
$$

$$
=(49.0 \mathrm{~N})(3.00 \mathrm{~m} / \mathrm{s})
$$

$$
=1.5 \times 10^{2} \mathrm{~W}
$$

6. $\quad$ power out $=\frac{W}{t}$ and $W=m g h$

$$
\text { power out }=\frac{m g h}{t}
$$

$$
\begin{aligned}
& =\frac{(20.0 \mathrm{~kg})\left(9.81 \mathrm{~m} / \mathrm{s}^{2}\right)(5.00 \mathrm{~m})}{3.50 \mathrm{~s}} \\
& =280 \mathrm{~W}
\end{aligned}
$$

$$
\text { efficiency }=\frac{\text { power out }}{\text { power in }} \times 100 \%
$$

$$
\begin{aligned}
& =\frac{280 \mathrm{~W}}{5.00 \times 10^{2} \mathrm{~W}} \times 100 \% \\
& =56.0 \%
\end{aligned}
$$

7. efficiency $=\frac{\text { power out }}{\text { power in }} \times 100 \%$

$$
\begin{aligned}
\text { power out } & =\frac{\text { efficiency } \times \text { power in }}{100 \%} \\
& =\frac{(82 \%)\left(1.00 \times 10^{5} \mathrm{~W}\right)}{100 \%} \\
& =\frac{(0.82)\left(1.00 \times 10^{5} \mathrm{~W}\right)}{1.00} \\
& =8.2 \times 10^{4} \mathrm{~W}
\end{aligned}
$$

$$
\text { power out }=\frac{W}{t} \text { and } W=m g h
$$

$$
\begin{aligned}
t & =\frac{m g h}{\text { power out }} \\
& =\frac{(50.0 \mathrm{~kg})\left(9.81 \mathrm{~m} / \mathrm{s}^{2}\right)(8.00 \mathrm{~m})}{8.2 \times 10^{4} \mathrm{~W}} \\
& =0.0479 \mathrm{~s}
\end{aligned}
$$

## Lesson 6-Machines and Efficiency

## PRACTICE EXERCISES

 ANSWERS AND SOLUTIONS1. $\quad$ work out $=m g h$

$$
\begin{aligned}
& =(225 \mathrm{~kg})\left(9.81 \mathrm{~m} / \mathrm{s}^{2}\right)(1.20 \mathrm{~m}) \\
& =2649 \mathrm{~J}
\end{aligned}
$$

work in $=F d$

$$
\begin{aligned}
& =(315 \mathrm{~N})(10.0 \mathrm{~m}) \\
& =3150 \mathrm{~J}
\end{aligned}
$$

efficiency $=\frac{\text { work out }}{\text { work in }} \times 100 \%$

$$
\begin{aligned}
& =\frac{2649 \mathrm{~J}}{3150 \mathrm{~J}} \times 100 \% \\
& =84.1 \%
\end{aligned}
$$

2. $\quad$ work out $=m g h$

$$
\begin{aligned}
& =(935 \mathrm{~N})(5.0 \mathrm{~m}) \\
& =4675 \mathrm{~J}
\end{aligned}
$$

work in $=F d$

$$
\begin{aligned}
& =(455 \mathrm{~N})(15.0 \mathrm{~m}) \\
& =6825 \mathrm{~J}
\end{aligned}
$$

efficiency $=\frac{\text { work out }}{\text { work in }} \times 100 \%$

$$
\begin{aligned}
& =\frac{4675 \mathrm{~J}}{6825 \mathrm{~J}} \times 100 \% \\
& =68 \%
\end{aligned}
$$

3. work out $=m g h$

$$
\begin{aligned}
& =(75.0 \mathrm{~kg})\left(9.81 \mathrm{~m} / \mathrm{s}^{2}\right)(3.0 \mathrm{~m}) \\
& =2207 \mathrm{~J}
\end{aligned}
$$

$$
\begin{aligned}
\text { efficiency } & =\frac{\text { work out }}{\text { work in }} \times 100 \% \\
\text { work in } & =\frac{2207 \mathrm{~J}}{78.5 \%} \times 100 \% \\
& =\frac{2205 \mathrm{~J}}{0.785} \times 1.00 \\
& =2809 \mathrm{~J}
\end{aligned}
$$

work in $=F d$

$$
\begin{aligned}
F & =\frac{\text { work in }}{d} \\
& =\frac{2809 \mathrm{~J}}{8.0 \mathrm{~m}} \\
& =3.5 \times 10^{2} \mathrm{~N}
\end{aligned}
$$

4. $\quad$ work out $=m g h$

$$
\begin{aligned}
& =(95 \mathrm{~kg})\left(9.81 \mathrm{~m} / \mathrm{s}^{2}\right)(0.11 \mathrm{~m}) \\
& =103 \mathrm{~J}
\end{aligned}
$$

efficiency $=\frac{\text { work out }}{\text { work in }} \times 100 \%$

$$
\text { work in }=\frac{103 \mathrm{~J}}{63 \%} \times 100 \%
$$

$$
\begin{aligned}
& =\frac{102 \mathrm{~J}}{0.63} \times 1.00 \\
& =163 \mathrm{~J}
\end{aligned}
$$

work in $=F d$

$$
\begin{aligned}
d & =\frac{\text { work in }}{F} \\
& =\frac{162 \mathrm{~J}}{262 \mathrm{~N}} \\
& =0.62 \mathrm{~m} \\
& =62 \mathrm{~cm}
\end{aligned}
$$

5. $\quad$ work out $=m g h$

$$
\begin{aligned}
& =(65.0 \mathrm{~kg})\left(9.81 \mathrm{~m} / \mathrm{s}^{2}\right)(1.92 \mathrm{~m}) \\
& =1224 \mathrm{~J} \\
\text { efficiency } & =\frac{\text { work out }}{\text { work in }} \times 100 \% \\
\text { work in } & =\frac{1224}{68.2 \%} \times 100 \% \\
& =\frac{1223 \mathrm{~J}}{0.682} \times 1.00 \\
& =1795 \mathrm{~J}
\end{aligned}
$$

Work required to overcome friction:

$$
\begin{aligned}
\mathrm{W} & =\text { work in }- \text { work out } \\
& =1795 \mathrm{~J}-1224 \mathrm{~J} \\
& =571 \mathrm{~J} \\
W & =F_{\mathrm{f}} d \\
& =\frac{571 \mathrm{~J}}{5.00 \mathrm{~m}} \\
& =114 \mathrm{~N}
\end{aligned}
$$

6. Although efficiency is defined in terms of work, it can expressed in terms of power.

$$
\begin{aligned}
\text { power out } & =\frac{W}{t} \text { and } W=m g h \\
\text { power out } & =\frac{m g h}{t} \\
& =\frac{\left(1.30 \times 10^{3} \mathrm{~kg}\right)\left(9.81 \mathrm{~m} / \mathrm{s}^{2}\right)(25.0 \mathrm{~m})}{60.0 \mathrm{~s}} \\
& =5.31 \times 10^{3} \mathrm{~W} \\
\text { efficiency } & =\frac{\text { power out }}{\text { power in }} \times 100 \% \\
& =\frac{5.31 \times 10^{3} \mathrm{~W}}{1.00 \times 10^{4} \mathrm{~W}} \times 100 \% \\
& =53.1 \%
\end{aligned}
$$

## Lesson 7-Electric Current

## PRACTICE EXERCISES

 ANSWERS AND SOLUTIONS1. $I=\frac{q}{t}$

$$
\begin{aligned}
q & =I t \\
& =(3.60 \mathrm{~A})(15.3 \mathrm{~s}) \\
& =55.1 \mathrm{C}
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{e}^{-} & =\frac{q}{1.60 \times 10^{-19} \mathrm{C}} \\
& =\frac{55.1 \mathrm{C}}{1.60 \times 10^{-19} \mathrm{C}} \\
& =3.44 \times 10^{20}
\end{aligned}
$$

2. $I=\frac{q}{t}$

$$
\begin{aligned}
t & =\frac{q}{I} \\
& =\frac{\left(2.0 \times 10^{20}\right)\left(1.60 \times 10^{-19} \mathrm{C}\right)}{10.0 \mathrm{~A}} \\
& =3.2 \mathrm{~s}
\end{aligned}
$$

3. $I=\frac{q}{t}$

$$
\begin{aligned}
& =\frac{5.60 \mathrm{C}}{15.4 \mathrm{~s}} \\
& =3.64 \times 10^{-1} \mathrm{~A}
\end{aligned}
$$

4. $\quad V=I R$

$$
\begin{aligned}
& =(8.00 \mathrm{~A})(12.0 \mathrm{~W}) \\
& =96.0 \mathrm{~V}
\end{aligned}
$$

5. $P=I^{2} R$

$$
\begin{aligned}
& =(11.0 \mathrm{~A})^{2}(7.20 \mathrm{~W}) \\
& =8.71 \times 10^{2} \mathrm{~W}
\end{aligned}
$$

$$
\begin{aligned}
P & =\frac{\Delta E}{t} \\
\Delta E & =P t \\
& =\left(8.71 \times 10^{2} \mathrm{~W}\right)(25.0 \mathrm{~s}) \\
& =2.18 \times 10^{4} \mathrm{~J}
\end{aligned}
$$

6. $P=\frac{\Delta E}{t}$

$$
\begin{aligned}
& =\frac{1.50 \times 10^{2} \mathrm{~J}}{5.50 \mathrm{~s}} \\
& =2.73 \times 10^{1} \mathrm{~W}
\end{aligned}
$$

$$
\begin{aligned}
P & =I^{2} R \\
R & =\frac{P}{I^{2}} \\
& =\frac{2.73 \times 10^{1} \mathrm{~W}}{(10.0 \mathrm{~A})^{2}} \\
& =2.73 \times 10^{-1} \Omega
\end{aligned}
$$

7. $P=I V$

$$
\begin{aligned}
I & =\frac{P}{V} \\
& =\frac{4.00 \times 10^{2} \mathrm{~W}}{1.20 \times 10^{2} \mathrm{~V}} \\
& =3.33 \mathrm{~A}
\end{aligned}
$$

8. a) $V=I R$

$$
\begin{aligned}
R & =\frac{V}{I} \\
& =\frac{1.20 \times 10^{2} \mathrm{~V}}{18.3 \mathrm{~A}} \\
& =6.56 \Omega
\end{aligned}
$$

b) power dissipated (or energy used per second)

$$
\begin{aligned}
P & =I V \\
& =(18.3 \mathrm{~A})\left(1.2 \times 10^{2} \mathrm{~V}\right) \\
& =2.20 \quad 10^{3} \mathrm{~J} / \mathrm{s}
\end{aligned}
$$

## Charge that passes through appliance

 each second$$
\begin{aligned}
q & =I t \\
& =(18.3 \mathrm{~A})(1.00 \mathrm{~s}) \\
& =18.3 \mathrm{C}
\end{aligned}
$$

$$
\begin{aligned}
\text { number of } \mathrm{e}^{-} & =\frac{\text { total charge }}{\text { charge on } 1 \mathrm{e}^{-}} \\
& =\frac{18.3 \mathrm{C}}{1.60 \times 10^{-19} \mathrm{C}} \\
& =1.14 \times 10^{20}
\end{aligned}
$$

Energy used per e ${ }^{-}$
$=\frac{\text { total energy used } / \text { second }}{\text { number of } \mathrm{e}^{-} \text {flowing } / \text { second }}$
$=\frac{2.20 \times 10^{3} \mathrm{~J} / \mathrm{s}}{1.14 \times 10^{20}}$
$=1.92 \times 10^{-17} \mathrm{~J}$
9. a) $V=I R$

$$
\begin{aligned}
& =(20.0 \mathrm{~A})(6.00 \Omega) \\
& =1.20 \times 10^{2} \mathrm{~V}
\end{aligned}
$$

b) $q=I t$

$$
\begin{aligned}
& =(20.0 \mathrm{~A})(60.0 \mathrm{~s}) \\
& =1200 \mathrm{C}
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{e}^{-} & =\frac{\text { total charge }}{\text { charge on } 1 \mathrm{e}^{-}} \\
& =\frac{1200 \mathrm{C}}{1.60 \times 10^{-19} \mathrm{C}} \\
& =7.50 \times 10^{21}
\end{aligned}
$$

10. a)

b) slope $=\frac{\text { rise }}{\text { run }}$

$$
\begin{aligned}
& =\frac{y_{2}-y_{1}}{x_{2}-x_{1}} \\
& =\frac{(0.70-0) \mathrm{A}}{(14.0-0) \mathrm{V}} \\
& =5.0 \times 10^{-2} \mathrm{~A} / \mathrm{V}
\end{aligned}
$$

$$
\begin{aligned}
R & =\frac{1}{\text { slope }} \\
& =\frac{1}{5.00 \times 10^{-2} \mathrm{~A} / \mathrm{V}} \\
& =(20.0 \pm 0.5) \Omega
\end{aligned}
$$

11. $P=\frac{\Delta E}{t}$
$\Delta E=P t$

$$
=(1.0 \mathrm{~kW})(5.0 \mathrm{~h})
$$

$$
=5.0 \mathrm{~kW} \cdot \mathrm{~h}
$$

$$
\begin{aligned}
\operatorname{cost} & =(\$ 0.060)(5.0 \mathrm{~kW} \cdot \mathrm{~h}) \\
& =\$ 0.30
\end{aligned}
$$

12. $P=\frac{\Delta E}{t}$
$\Delta E=P t$

$$
=\left(6.00 \times 10^{2} \mathrm{~W}\right)\left(14 \mathrm{~h} \times 3.6 \times 10^{3} \mathrm{~s} / \mathrm{h}\right)
$$

$$
=3.0 \times 10^{7} \mathrm{~J}
$$

13. $W=m g h$

$$
\begin{aligned}
& =(45 \mathrm{~kg})\left(9.81 \mathrm{~m} / \mathrm{s}^{2}\right)(9.0 \mathrm{~m}) \\
& =3973 \mathrm{~J}
\end{aligned}
$$

$25 \%$ of $7.5 \times 10^{2} \mathrm{~W}=187.5 \mathrm{~W}$

$$
\begin{aligned}
P & =\frac{W}{t} \\
t & =\frac{W}{P} \\
& =\frac{3973 \mathrm{~J}}{187.5 \mathrm{~W}} \\
& =21 \mathrm{~s}
\end{aligned}
$$

## Lesson 8-Electric Circuits <br> PRACTICE EXERCISES ANSWERS AND SOLUTIONS

1. In a series circuit:
$I=I_{1}=I_{2}=I_{3}$
$I_{1}=1.7 \mathrm{~A}$
$I_{2}=1.7 \mathrm{~A}$
$I_{3}=1.7 \mathrm{~A}$
2. In a parallel circuit:

$$
\begin{aligned}
I & =I_{1}+I_{2} \\
I & =2.1 \mathrm{~A}+1.5 \mathrm{~A} \\
& =3.6 \mathrm{~A}
\end{aligned}
$$

Consider the series circuit:

$$
I=I_{3}
$$

$\therefore I_{3}=3.6 \mathrm{~A}$
3. In a series circuit:

$$
\begin{aligned}
V & =V_{1}+V_{2} \\
12.0 \mathrm{~V} & =8.0 \mathrm{~V}+V_{2} \\
V_{2} & =4.0 \mathrm{~V}
\end{aligned}
$$

4. In a parallel circuit:

$$
\begin{aligned}
V & =V_{1}=V_{2} \\
20.0 \mathrm{~V} & =20.0 \mathrm{~V}=V_{2} \\
\therefore \quad V_{2} & =20.0 \mathrm{~V}
\end{aligned}
$$

5. At first consider the series circuit:

$$
\begin{aligned}
V & =V_{1}+V_{2} \\
45.0 \mathrm{~V} & =11.0 \mathrm{~V}+V_{2} \\
V_{2} & =45.0 \mathrm{~V}-11.0 \mathrm{~V} \\
& =34.0 \mathrm{~V}
\end{aligned}
$$

Then consider the parallel circuit:

$$
\begin{aligned}
V_{2} & =V_{3} \\
34.0 \mathrm{~V} & =V_{2}=V_{3} \\
\therefore V_{3} & =34.0 \mathrm{~V}
\end{aligned}
$$

6. In a series circuit:

$$
\begin{aligned}
R_{\mathrm{eq}} & =R_{1}+R_{2} \\
& =15.0 \Omega+20.0 \Omega \\
& =35.0 \Omega
\end{aligned}
$$

7. In a parallel circuit:

$$
\begin{aligned}
\frac{1}{R_{\mathrm{eq}}} & =\frac{1}{R_{1}}+\frac{1}{R_{2}} \\
& =\frac{1}{6.0 \Omega}+\frac{1}{8.0 \Omega} \\
R_{\mathrm{eq}} & =3.4 \Omega
\end{aligned}
$$

8. Find equivalent resistance in the two parallel resistors first:

$$
\begin{aligned}
\frac{1}{R_{\mathrm{eq}(\mathrm{p})}} & =\frac{1}{R_{2}}+\frac{1}{R_{3}} \\
& =\frac{1}{3.0 \Omega}+\frac{1}{6.0 \Omega} \\
R_{\mathrm{eq}(\mathrm{p})} & =2.0 \Omega
\end{aligned}
$$

Now, add in the series resistors:

$$
\begin{aligned}
R_{\mathrm{eq}(\mathrm{~s})} & =R_{1}+\mathrm{R}_{\mathrm{eq}(\mathrm{p})} \\
& =2.0 \Omega+2.0 \Omega \\
& =4.0 \Omega
\end{aligned}
$$

9. In a series circuit:

$$
\begin{aligned}
R_{\mathrm{eq}} & =R_{1}+R_{2}+R_{3} \\
& =9.0 \Omega+3.0 \Omega+12.0 \Omega \\
& =24.0 \Omega
\end{aligned}
$$

10. In a parallel circuit:

$$
\begin{aligned}
\frac{1}{R_{\mathrm{eq}}} & =\frac{1}{R_{1}}+\frac{1}{R_{2}}+\frac{1}{R_{3}} \\
& =\frac{1}{2.0 \Omega}+\frac{1}{4.0 \Omega}+\frac{1}{8.0 \Omega} \\
R_{\mathrm{eq}} & =1.1 \Omega
\end{aligned}
$$

11. In a parallel circuit:

$$
\begin{aligned}
\frac{1}{R_{\mathrm{eq}}} & =\frac{1}{R_{1}}+\frac{1}{R_{2}}+\frac{1}{R_{3}} \\
\frac{1}{2.0 \Omega} & =\frac{1}{4.5 \Omega}+\frac{1}{9.0 \Omega}+\frac{1}{R_{3}} \\
\frac{1}{R_{3}} & =\frac{1}{2.0 \Omega}-\left(\frac{1}{4.5 \Omega}+\frac{1}{9.0 \Omega}\right) \\
R_{3} & =6.0 \Omega
\end{aligned}
$$

12. In a series circuit:

$$
\begin{aligned}
R_{\mathrm{eq}} & =R_{1}+R_{2}+R_{3} \\
12.0 \Omega & =6.0 \Omega+4.0 \Omega+R_{3} \\
\therefore R_{3} & =12.0 \Omega-10.0 \Omega \\
& =2.0 \Omega
\end{aligned}
$$

13. a) In a series circuit:

$$
\begin{aligned}
R_{\mathrm{eq}} & =R_{1}+R_{2} \\
& =10.0 \Omega+15.0 \Omega \\
& =25.0 \Omega
\end{aligned}
$$

$$
I=\frac{V}{R_{\mathrm{eq}}}
$$

$$
=\frac{30.0 \mathrm{~V}}{25.0 \Omega}
$$

$$
=1.20 \mathrm{~A}
$$

$$
I=I_{1}=I_{2}
$$

$$
\therefore I_{1}=1.20 \mathrm{~A}
$$

$$
I_{2}=1.20 \mathrm{~A}
$$

b) $\quad P=I^{2} R_{1}$

$$
\begin{aligned}
& =(1.20 \mathrm{~A})^{2}(10.0 \Omega) \\
& =14.4 \mathrm{~W}
\end{aligned}
$$

14. a) In a parallel circuit:

$$
\begin{aligned}
\frac{1}{R_{\mathrm{eq}}} & =\frac{1}{R_{1}}+\frac{1}{R_{2}} \\
& =\frac{1}{6.0 \Omega}+\frac{1}{3.0 \Omega} \\
R_{\mathrm{eq}} & =2.0 \Omega
\end{aligned}
$$

$$
V=15.0 \mathrm{~V}
$$

$$
I=\frac{V}{R_{\mathrm{eq}}}
$$

$$
=\frac{15.0 \mathrm{~V}}{2.0 \Omega}
$$

$$
=7.5 \mathrm{~A}
$$

b) $\quad I_{1}=\frac{7.5 \mathrm{~A}}{3}$

$$
=2.5 \mathrm{~A}
$$

$$
\begin{aligned}
P & =I^{2} R \\
& =(2.5 \mathrm{~A})^{2}(6.0 \Omega) \\
& =38 \mathrm{~W}
\end{aligned}
$$

15. a) In a parallel circuit:

$$
\begin{aligned}
\frac{1}{R_{\mathrm{eq}}} & =\frac{1}{R_{1}}+\frac{1}{R_{2}} \\
& =\frac{1}{6.0 \Omega}+\frac{1}{8.0 \Omega} \\
R_{\mathrm{eq}} & =3.43 \Omega
\end{aligned}
$$

In the second circuit:

$$
\begin{aligned}
I_{2} & =\frac{V_{2}}{R_{2}} \\
& =\frac{12.0 \mathrm{~V}}{8.0 \Omega} \\
& =1.5 \mathrm{~A} \\
I_{1} & =\frac{V}{R_{\mathrm{eq}}} \\
& =\frac{12.0 \mathrm{~V}}{3.43 \Omega} \\
& =3.5 \mathrm{~A}
\end{aligned}
$$

b) $\quad P=I^{2} R$

$$
\begin{aligned}
& =(3.5 \mathrm{~A})^{2}(3.43 \Omega) \\
& =42 \mathrm{~W}
\end{aligned}
$$

16. a) Find equivalent resistance in the two parallel resistors first:

$$
\begin{aligned}
\frac{1}{R_{\mathrm{eq}(\mathrm{p})}} & =\frac{1}{R_{2}}+\frac{1}{R_{3}} \\
& =\frac{1}{2.0 \Omega}+\frac{1}{4.0 \Omega} \\
R_{\mathrm{eq}(\mathrm{p})} & =1.33 \Omega
\end{aligned}
$$

In the total circuit:

$$
\begin{aligned}
R_{\mathrm{eq}(\mathrm{~s})} & =R_{\mathrm{T}(\mathrm{p})}+R_{1} \\
& =1.33 \Omega+6.0 \Omega \\
& =7.33 \Omega
\end{aligned}
$$

$$
\begin{aligned}
I_{1} & =I_{3} \\
& =\frac{V}{R_{\text {eq(s) }}} \\
& =\frac{20.0 \mathrm{~V}}{7.33 \Omega} \\
& =2.7 \mathrm{~A}
\end{aligned}
$$

To find the proportion of this current that passes through $R_{3}$, we will use a ratio:
$I=\frac{V}{R}$ or $I \alpha \frac{1}{R}$
$\frac{R_{3}}{R_{2}}=\frac{4.0 \Omega}{2.0 \Omega}=2.0$

If the resistance $R_{3}$ is 2.0 times the resistance $R_{2}$, then the current through $R_{3}$ is 0.5 times the current through $R_{2}$.
$I_{\mathrm{T}}=I_{\mathrm{R} 2}+I_{\mathrm{R} 3}$

Let $I_{2}=$ current through $R_{3}$.

$$
\begin{aligned}
\therefore 2.73 \mathrm{~A} & =2 I_{2}+I_{2} \\
3 I_{2} & =2.7 \mathrm{~A} \\
I_{2} & =\frac{2.7 \mathrm{~A}}{3} \\
& =0.91 \mathrm{~A}
\end{aligned}
$$

b) $\quad P=I^{2} R$

$$
\begin{aligned}
& =(2.73 \mathrm{~A})^{2}(7.33 \Omega) \\
& =55 \mathrm{~W}
\end{aligned}
$$

17. a) In a series circuit:

$$
\begin{aligned}
R_{\mathrm{eq}} & =R_{1}+R_{2}+R_{3} \\
& =2.0 \Omega+2.5 \Omega+3.0 \Omega \\
& =7.5 \Omega \\
V & =I R \\
& =I R_{\mathrm{eq}} \\
& =(8.0 \mathrm{~A})(7.5 \Omega) \\
& =6.0 \times 10^{1} \mathrm{~V}
\end{aligned}
$$

b) $\quad P=I^{2} R$

$$
\begin{aligned}
& =(8.0 \mathrm{~A})^{2}(7.5 \mathrm{~W}) \\
& =4.8 \times 10^{2} \mathrm{~W}
\end{aligned}
$$

18. a) In a parallel circuit:

$$
\begin{aligned}
\frac{1}{R_{\mathrm{eq}}} & =\frac{1}{R_{1}}+\frac{1}{R_{2}} \\
& =\frac{1}{8.0 \Omega}+\frac{1}{10.0 \Omega} \\
R_{\mathrm{eq}} & =4.44 \Omega \\
I & =\frac{V}{R} \\
& =\frac{V}{R_{\mathrm{eq}}} \\
& =\frac{25.0 \mathrm{~V}}{4.44 \Omega} \\
& =5.6 \mathrm{~A}
\end{aligned}
$$

b) $\quad P=I^{2} R$

$$
\begin{aligned}
& =(5.63 \mathrm{~A})^{2}(4.44 \Omega) \\
& =1.410^{2} \mathrm{~W}
\end{aligned}
$$

19. $I_{1}=\frac{V}{R_{\text {eq(total) }}}$

$$
\begin{aligned}
R_{\text {eqt(total) }} & =\frac{V}{I_{\mathrm{eq}}} \\
& =\frac{90.0 \mathrm{~V}}{4.5 \mathrm{~A}} \\
& =20.0 \Omega
\end{aligned}
$$

Add $R_{1}$ and $R_{2}$ for parallel circuit I:

$$
\begin{aligned}
& \frac{1}{R_{\mathrm{eq}}^{\mathrm{I}}}=\frac{1}{20.0 \Omega}+\frac{1}{10.0 \Omega} \\
& R_{\mathrm{eq}}^{\mathrm{I}}=6.67 \Omega
\end{aligned}
$$

Add $R_{4}$ and $R_{5}$ for parallel circuit II:
$\frac{1}{R^{\mathrm{II}}{ }_{\text {eq }}}=\frac{1}{20.0 \Omega}+\frac{1}{10.0 \Omega}$
$R^{\text {II }}{ }_{\text {eq }}=6.67 \Omega$

Find $R_{3}$ :
In a series circuit:

$$
\begin{aligned}
R_{\text {eqtotoal) }} & =R_{\mathrm{eq}}^{\mathrm{I}}+R_{\mathrm{eq}}^{\mathrm{II}}+R_{3} \\
R_{3} & =R_{\mathrm{eq}(\mathrm{total})}-\left(R_{\mathrm{eq}}^{\mathrm{I}}+R_{\mathrm{eq}}^{\mathrm{II}}\right) \\
& =20.0 \Omega-(6.67 \Omega+6.67 \Omega) \\
& =6.7 \Omega
\end{aligned}
$$

Now, find the current:
$I_{3}=4.5 \mathrm{~A}$

The sum of the current through $R_{1}$ and $R_{2}$ is also 4.5 A .

Since $I \alpha \frac{1}{R}$, the current through $R_{1}$ will be 0.50 the current through $R_{2}$.

Let $I_{R_{2}}=I_{2}$

$$
\begin{aligned}
I_{\mathrm{T}\left(\text { through } R_{1} \text { and } R_{2}\right)} & =I_{R_{1}}+I_{R_{2}} \\
4.5 \mathrm{~A} & =0.50 I_{2}+I_{2} \\
4.5 \mathrm{~A} & =1.50 I_{2} \\
I_{2} & =1.5 \mathrm{~A}
\end{aligned}
$$

In this problem:
$I_{4}=I_{2}=1.5 \mathrm{~A}$
20. a) Find the resistance of each coil:

$$
\begin{aligned}
P & =\frac{V^{2}}{R} \\
R & =\frac{V^{2}}{P} \\
& =\frac{(120 \mathrm{~V})^{2}}{6.0 \times 10^{-2} \mathrm{~W}} \\
& =24 \Omega
\end{aligned}
$$

Total resistance of the heater:

$$
\begin{aligned}
R_{\mathrm{eq}} & =R_{1}+R_{2} \\
& =24 \Omega+24 \Omega \\
& =48 \Omega
\end{aligned}
$$

Power consumed by heater:

$$
\begin{aligned}
P & =\frac{V^{2}}{R} \\
& =\frac{(120 \mathrm{~V})^{2}}{48 \Omega} \\
& =300 \mathrm{~W} \text { or } 0.30 \mathrm{~kW}
\end{aligned}
$$

Time per day:

$$
\begin{aligned}
P & =\frac{\Delta E}{t} \\
t & =\frac{\Delta E}{P} \\
& =\frac{1.5 \times 10^{7} \mathrm{~J}}{300 \mathrm{~W}} \\
& =5.0 \times 10^{4} \mathrm{~s}
\end{aligned}
$$

Hours per day:

$$
\begin{aligned}
\# \text { of hours } & =\frac{5.0 \times 10^{4} \mathrm{~s}}{3600 \mathrm{~s} / \mathrm{h}} \\
& =14 \mathrm{~h}
\end{aligned}
$$

b) Find the resistance of each coil:

$$
\begin{aligned}
P & =\frac{V^{2}}{R} \\
R & =\frac{V^{2}}{P} \\
& =\frac{(120 \mathrm{~V})^{2}}{6.0 \times 10^{-2} \mathrm{~W}} \\
& =24 \Omega
\end{aligned}
$$

Total resistance of the heater:
$\frac{1}{R_{\text {eq }}}=\frac{1}{R_{1}}+\frac{1}{R_{2}}$
$=\frac{1}{24 \Omega}+\frac{1}{24 \Omega}$
$R_{\text {eq }}=12 \Omega$
$P=\frac{V^{2}}{R}$
$=\frac{(120 \mathrm{~V})^{2}}{12 \Omega}$
$=1200 \mathrm{~W}$ or 1.2 kW

Time per day:

$$
\begin{aligned}
P & =\frac{\Delta E}{t} \\
t & =\frac{\Delta E}{P} \\
& =\frac{1.5 \times 10^{7} \mathrm{~J}}{1200 \mathrm{~W}} \\
& =1.25 \times 10^{4} \mathrm{~s}
\end{aligned}
$$

Hours per day:

$$
\begin{aligned}
\# \text { of hours } & =\frac{1.25 \times 10^{4} \mathrm{~s}}{3600 \mathrm{~s} / \mathrm{h}} \\
& =3.5 \mathrm{~h}
\end{aligned}
$$

c) There would be no cost advantage between the two arrangements. Each case uses 4.2 kWh of electricity.

## ALTERNATE METHOD

a) Find the resistance of each coil:

$$
\begin{aligned}
P & =\frac{V^{2}}{R} \\
R & =\frac{V^{2}}{P} \\
& =\frac{(120 \mathrm{~V})^{2}}{6.0 \times 10^{-2} \mathrm{~W}} \\
& =24 \Omega
\end{aligned}
$$

Power dissipated in each coil:

$$
\begin{aligned}
P & =\frac{V^{2}}{R} \\
& =\frac{(60 \mathrm{~V})^{2}}{24 \Omega} \\
& =150 \mathrm{~W}
\end{aligned}
$$

Power dissipated by heater:

$$
2 \times 150 \mathrm{~W}=300 \mathrm{~W}
$$

Time:

$$
\begin{aligned}
P & =\frac{\Delta E}{t} \\
t & =\frac{\Delta E}{P} \\
& =\frac{1.5 \times 10^{7} \mathrm{~J}}{300 \mathrm{~W}} \\
& =5.0 \times 10^{4} \mathrm{~s}
\end{aligned}
$$

Hours:

$$
\begin{aligned}
\# \text { of hours } & =\frac{5.0 \times 10^{4} \mathrm{~s}}{3600 \mathrm{~s} / \mathrm{h}} \\
& =14 \mathrm{~h}
\end{aligned}
$$

b) Find the resistance of each coil:

$$
\begin{aligned}
P & =\frac{V^{2}}{R} \\
R & =\frac{V^{2}}{P} \\
& =\frac{(120 \mathrm{~V})^{2}}{6.0 \times 10^{-2} \mathrm{~W}} \\
& =24 \Omega
\end{aligned}
$$

Power dissipated by each coil:

$$
\begin{aligned}
P & =\frac{V^{2}}{R} \\
& =\frac{(120 \mathrm{~V})^{2}}{24 \Omega} \\
& =600 \mathrm{~W}
\end{aligned}
$$

Power dissipated by heater:
$2 \times 600 \mathrm{~W}=1200 \mathrm{~W}$
Time:

$$
\begin{aligned}
P & =\frac{\Delta E}{t} \\
t & =\frac{\Delta E}{P} \\
& =\frac{1.5 \times 10^{7} \mathrm{~J}}{1200 \mathrm{~W}} \\
& =1.25 \times 10^{4} \mathrm{~s}
\end{aligned}
$$

Hours per day:

$$
\begin{aligned}
\# \text { of hours } & =\frac{1.25 \times 10^{4} \mathrm{~s}}{3600 \mathrm{~s} / \mathrm{h}} \\
& =3.5 \mathrm{~h}
\end{aligned}
$$

21. Find total resistance of the circuit:


Add $R_{1}$ and $R_{2}$

$$
\begin{aligned}
\frac{1}{R_{\mathrm{eq}(1)}} & =\frac{1}{R_{1}}+\frac{1}{R_{2}} \\
& =\frac{1}{12 \Omega}+\frac{1}{12 \Omega} \\
\mathrm{R}_{\mathrm{eq}(1)} & =6.0 \Omega
\end{aligned}
$$

Add $6.0 \Omega$ to $R_{3}$

$$
\begin{aligned}
R_{\mathrm{eq}(2)} & =6.0 \mathrm{~W}+12 \mathrm{~W} \\
& =18 \mathrm{~W}
\end{aligned}
$$

Add $18 \Omega$ to $R_{4}$
$\frac{1}{R_{\mathrm{eq}(3)}}=\frac{1}{12 \Omega}+\frac{1}{18 \Omega}$
$\mathrm{R}_{\mathrm{eq}(3)}=7.2 \Omega$

Find power dissipated by the circuit:

$$
\begin{aligned}
P & =\frac{V^{2}}{R} \\
& =\frac{(120 \mathrm{~V})^{2}}{7.2 \Omega} \\
& =2.0 \times 10^{3} \mathrm{~W}
\end{aligned}
$$

Find energy used in 4.0 h

$$
\begin{aligned}
P & =\frac{\Delta E}{t} \\
\Delta E & =P t \\
& =\left(2.0 \times 10^{4} \mathrm{~W}\right)(4.0 \mathrm{~h} \times 3600 \mathrm{~s} / \mathrm{h}) \\
& =2.9 \times 10^{7} \mathrm{~J}
\end{aligned}
$$

Use: $\Delta E=m c \Delta T$

$$
\begin{aligned}
\Delta T & =\frac{\Delta E}{m C} \\
& =\frac{2.9 \times 10^{7} \mathrm{~J}}{(200 \mathrm{~kg})\left(4.19 \times 10^{3} \mathrm{~J} / \mathrm{kg} \cdot \mathrm{C}\right)} \\
& =34^{\circ} \mathrm{C}
\end{aligned}
$$

Final temperature:

$$
\begin{aligned}
T_{\mathrm{f}} & =T_{\mathrm{i}}+\Delta T \\
& =15^{\circ} \mathrm{C}+34^{\circ} \mathrm{C} \\
& =49^{\circ} \mathrm{C}
\end{aligned}
$$

22. a)

$$
\begin{aligned}
V & =I R \\
R & =\frac{V}{I} \\
& =\frac{120 \mathrm{~V}}{30.0 \mathrm{~A}}=4.0 \Omega \\
& =\frac{120 \mathrm{~V}}{20.0 \mathrm{~A}}=6.0 \Omega \\
& =\frac{120 \mathrm{~V}}{15.0 \mathrm{~A}}=8.0 \Omega \\
& =\frac{120 \mathrm{~V}}{10.0 \mathrm{~A}}=12 \Omega \\
& =\frac{120 \mathrm{~V}}{5.0 \mathrm{~A}}=24 \Omega
\end{aligned}
$$

b)

c)


$$
\begin{aligned}
\frac{1}{A} & =\frac{1}{2.1 \times 10^{-9} \mathrm{~m}^{2}} \\
& =4.8 \times 10^{8} / \mathrm{m}^{2} \\
& =4.8 \\
\frac{1}{A} & =\frac{1}{1.4 \times 0^{-9} \mathrm{~m}^{2}} \\
& =7.1 \times 10^{8} / \mathrm{m}^{2} \\
& =7.1 \\
\frac{1}{A} & =\frac{1}{1.1 \times 10^{-9} \mathrm{~m}^{2}} \\
& =9.1 \times 10^{8} / \mathrm{m}^{2} \\
& =9.1 \\
\frac{1}{A} & =\frac{1}{0.70 \times 10^{-9} \mathrm{~m}^{2}} \\
& =14.3 \times 10^{8} / \mathrm{m}^{2} \\
& =14 \\
\frac{1}{A} & =\frac{1}{0.35 \times 10^{-9} \mathrm{~m}^{2}} \\
& =28.6 \times 10^{8} / \mathrm{m}^{2} \\
& =29
\end{aligned}
$$

## Lesson 9-Electromotive Force (EMF)

## PRACTICE EXERCISES

 ANSWERS AND SOLUTIONS1. $V=\varepsilon-I r$

$$
\begin{aligned}
& =1.5 \mathrm{~V}-(1.0 \mathrm{~A})(0.50 \Omega) \\
& =1.5 \mathrm{~V}-0.50 \mathrm{~V} \\
& =1.0 \mathrm{~V}
\end{aligned}
$$

2. $V=\varepsilon-I r$
$\begin{aligned} \mathcal{E} & =V+I r \\ & =5.0 \mathrm{~V}+(1.2 \mathrm{~A})(0.72 \Omega) \\ & =5.9 \mathrm{~V}\end{aligned}$

$$
=5.9 \mathrm{~V}
$$

3. Current flowing to battery

$$
\begin{aligned}
V & =\varepsilon+I r \\
& =24 \mathrm{~V}+(24 \mathrm{~A})(0.25 \Omega) \\
& =30 \mathrm{~V}
\end{aligned}
$$

4. $V=\mathcal{E}-I r$

$$
\begin{aligned}
I R & =\mathcal{E}-I r \\
I(R+r) & =\mathcal{E} \\
I & =\frac{\mathcal{E}}{(R+r)} \\
& =\frac{12 \mathrm{~V}}{4.0 \Omega+1.0 \Omega} \\
& =\frac{12 \mathrm{~V}}{5.0 \Omega}=2.4 \Omega
\end{aligned}
$$

5. $V=\mathcal{E}-I r$

$$
\begin{aligned}
r & =\frac{\varepsilon-V}{I} \\
& =\frac{120 \mathrm{~V}-115 \mathrm{~V}}{12 \mathrm{~A}} \\
& =0.42 \Omega
\end{aligned}
$$

## Lesson 10-Thermal Energy

## PRACTICE EXERCISES ANSWERS AND SOLUTIONS

1. $\Delta E_{\mathrm{h}}=m \Delta T c$
$=(0.462 \mathrm{~kg})\left(80^{\circ} \mathrm{C}-24^{\circ} \mathrm{C}\right)\left(4.18 \times 10^{3} \mathrm{~J} /\left(\mathrm{kg} \cdot{ }^{\circ} \mathrm{C}\right)\right)$
$=1.08 \times 10^{5} \mathrm{~J}$
2. $\Delta E_{\mathrm{h}}=m \Delta T c$

$$
\begin{aligned}
& =(0.462 \mathrm{~kg})\left(80^{\circ} \mathrm{C}-24^{\circ} \mathrm{C}\right)\left(1.30 \times 10^{2} \mathrm{~J} /\left(\mathrm{kg} \cdot{ }^{\circ} \mathrm{C}\right)\right) \\
& =3.36 \times 10^{3} \mathrm{~J}
\end{aligned}
$$

3. $\Delta E_{\mathrm{h}}=m \Delta T c$

$$
\begin{aligned}
\Delta T & =\frac{\Delta E_{\mathrm{h}}}{m c} \\
& =\frac{2.50 \times 10^{4} \mathrm{~J}}{(0.200 \mathrm{~kg})\left(3.47 \times 10^{3} \mathrm{~J} /\left(\mathrm{kg} \cdot{ }^{\circ} \mathrm{C}\right)\right)} \\
& =36.0^{\circ} \mathrm{C}
\end{aligned}
$$

$\Delta T=T_{\mathrm{f}}-T_{\mathrm{i}}$
$T_{\mathrm{f}}=T_{\mathrm{i}}+\Delta T$

$$
=20.0^{\circ} \mathrm{C}+36.0^{\circ} \mathrm{C}
$$

$$
=56.0^{\circ} \mathrm{C}
$$

4. $\Delta E_{\mathrm{h}}=m \Delta T c$

$$
\begin{aligned}
m & =\frac{\Delta E_{\mathrm{h}}}{\Delta T c} \\
& =\frac{2.10 \times 10^{4} \mathrm{~J}}{\left(15.0^{\circ} \mathrm{C}\right)\left(4.18 \times 10^{3} \mathrm{~J} /(\mathrm{kg} \cdot \mathrm{C})\right)} \\
& =0.335 \mathrm{~kg}
\end{aligned}
$$

5. heat gained by cold water + heat lost by hot water $=0$

$$
\begin{aligned}
& \Delta E_{\mathrm{hC}}+\Delta E_{\mathrm{hH}}=0 \\
& \Delta E_{\mathrm{hC}}=-\Delta E_{\mathrm{hH}} \\
& m_{\mathrm{C}} \Delta T_{\mathrm{C}} c_{\mathrm{C}}=-m_{\mathrm{H}} \Delta T_{\mathrm{H}} c_{\mathrm{H}} \\
& (0.185 \mathrm{~kg})\left(T_{\mathrm{f}}-12.0^{\circ} \mathrm{C}\right)\left(4.18 \times 10^{3} \mathrm{~J} /\left(\mathrm{kg} \cdot{ }^{\circ} \mathrm{C}\right)\right) \\
& =-(0.295 \mathrm{~kg})\left(T_{\mathrm{f}}-85.0^{\circ} \mathrm{C}\right)\left(4.18 \times 10^{3} \mathrm{~J} /\left(\mathrm{kg} \cdot{ }^{\circ} \mathrm{C}\right)\right) \\
& \left(7.733 \times 10^{2} \mathrm{~J} /{ }^{\circ} \mathrm{C}\right)\left(T_{\mathrm{f}}-12.0^{\circ} \mathrm{C}\right) \\
& =-\left(1.233 \times 10^{3} \mathrm{~J} /{ }^{\circ} \mathrm{C}\right)\left(T_{\mathrm{f}}-85.0^{\circ} \mathrm{C}\right) \\
& \left(7.733 \times 10^{2} \mathrm{~J} /{ }^{\circ} \mathrm{C}\right) T_{\mathrm{f}}-9.280 \times 10^{3} \mathrm{~J} \\
& =-\left(1.233 \times 10^{3} \mathrm{~J} /{ }^{\circ} \mathrm{C}\right) T_{\mathrm{f}}+1.048 \times 10^{5} \mathrm{~J} \\
& \begin{array}{r}
\left(7.733 \times 10^{2} \mathrm{~J} /{ }^{\circ} \mathrm{C}\right) T_{\mathrm{f}}+\left(1.233 \times 10^{3} \mathrm{~J} /{ }^{\circ} \mathrm{C}\right) T_{\mathrm{f}} \\
=1.048 \times 10^{5} \mathrm{~J}+9.280 \times 10^{3} \mathrm{~J}
\end{array} \\
& \begin{array}{r}
\left(2.006 \times 10^{3} \mathrm{~J} /{ }^{\circ} \mathrm{C}\right) T_{\mathrm{f}}=1.14 \times 10^{5} \mathrm{~J} \\
T_{\mathrm{f}}=\frac{1.140 \times 10^{5} \mathrm{~J}}{2.006 \times 10^{3} \mathrm{~J} /{ }^{\circ} \mathrm{C}} \\
\quad=56.8{ }^{\circ} \mathrm{C}
\end{array}
\end{aligned}
$$

6. heat gained by water + heat lost by copper $=0$

$$
\begin{aligned}
& \Delta E_{\mathrm{hW}}+\Delta E_{\mathrm{hC}}=0 \\
& \Delta E_{\mathrm{hW}}=-\Delta E_{\mathrm{hC}} \\
& m_{\mathrm{w}} \Delta T_{\mathrm{W}} c_{\mathrm{W}}=-m_{\mathrm{C}} \Delta T_{\mathrm{C}} c_{\mathrm{C}} \\
& (0.275 \mathrm{~kg})\left(T_{\mathrm{f}}-12.0^{\circ} \mathrm{C}\right)\left(4.18 \times 10^{3} \mathrm{~J} /\left(\mathrm{kg} \cdot{ }^{\circ} \mathrm{C}\right)\right) \\
& =-(0.240 \mathrm{~kg})\left(T_{\mathrm{f}}-215^{\circ} \mathrm{C}\right)\left(3.90 \times 10^{2} \mathrm{~J} /\left(\mathrm{kg} \cdot{ }^{\circ} \mathrm{C}\right)\right) \\
& \left(1.150 \times 10^{3} \mathrm{~J} /{ }^{\circ} \mathrm{C}\right)\left(T_{\mathrm{f}}-12.0^{\circ} \mathrm{C}\right) \\
& =-\left(93.60 \mathrm{~J} /{ }^{\circ} \mathrm{C}\right)\left(T_{\mathrm{f}}-215^{\circ} \mathrm{C}\right) \\
& \left(1.150 \times 10^{3} \mathrm{~J} /{ }^{\circ} \mathrm{C}\right) T_{\mathrm{f}}-1.380 \times 10^{4} \mathrm{~J} \\
& \quad=-\left(93.60 \mathrm{~J} /{ }^{\circ} \mathrm{C}\right) T_{\mathrm{f}}+2.012 \times 10^{4} \mathrm{~J} \\
& \begin{array}{r}
\left(1.150 \times 10^{3} \mathrm{~J} /{ }^{\circ} \mathrm{C}\right) T_{\mathrm{f}}
\end{array}+\left(93.60 \mathrm{~J} /{ }^{\circ} \mathrm{C}\right) T_{\mathrm{f}} \\
& \quad=1.380 \times 10^{4} \mathrm{~J}+2.012 \times 10^{4} \mathrm{~J} \\
& \left(1.244 \times 10^{3} \mathrm{~J} /{ }^{\circ} \mathrm{C}\right) T_{\mathrm{f}}=3.392 \times 10^{4} \mathrm{~J} \\
& T_{\mathrm{f}}=\frac{3.392 \times 10^{4} \mathrm{~J}}{1.244 \times 10^{3} \mathrm{~J} /{ }^{\circ} \mathrm{C}} \\
& T_{\mathrm{f}}=27.3^{\circ} \mathrm{C}
\end{aligned}
$$

7. heat gained by water + heat lost by metal $=0$

$$
\begin{aligned}
& \Delta E_{\mathrm{hW}}+\Delta E_{\mathrm{hM}}=0 \\
& \Delta E_{\mathrm{hW}}=-\Delta E_{\mathrm{hM}} \\
& m_{\mathrm{w}} \Delta T_{\mathrm{w}} c_{\mathrm{w}}=-m_{\mathrm{M}} \Delta T_{\mathrm{M}} c_{\mathrm{M}} \\
&(0.265 \mathrm{~kg})\left(33.0^{\circ} \mathrm{C}-26.0^{\circ} \mathrm{C}\right)\left(4.18 \times 10^{3} \mathrm{~J} /\left(\mathrm{kg} \cdot{ }^{\circ} \mathrm{C}\right)\right) \\
&=-(0.352 \mathrm{~kg})\left(33.0^{\circ} \mathrm{C}-215^{\circ} \mathrm{C}\right) c_{\mathrm{M}} \\
& 7.754 \times 10^{3} \mathrm{~J}=\left(64.06 \mathrm{~kg} \cdot{ }^{\circ} \mathrm{C}\right) c_{\mathrm{M}} \\
& c_{\mathrm{M}}=1.21 \times 10^{2} \mathrm{~J} /\left(\mathrm{kg} \cdot{ }^{\circ} \mathrm{C}\right)
\end{aligned}
$$

8. $E_{\mathrm{k}}=\frac{1}{2} m v^{2}$

$$
\begin{aligned}
& =\frac{1}{2}(0.001 \mathrm{~kg})(40.0 \mathrm{~m} / \mathrm{s})^{2} \\
& =0.8 \mathrm{~J} \\
\Delta E_{\mathrm{h}} & =m \Delta T c \\
\Delta T & =\frac{\Delta E_{\mathrm{h}}}{m c} \\
& =\frac{0.8 \mathrm{~J}}{(0.101 \mathrm{~kg})\left(4.18 \times 10^{3} \mathrm{~J} /\left(\mathrm{kg} \cdot{ }^{\circ} \mathrm{C}\right)\right)} \\
& =1.89 \times 10^{-30} \mathrm{C}
\end{aligned}
$$

9. heat gained by cold liquid + heat lost by warm liquid $=0$

$$
\begin{aligned}
& \Delta E_{\mathrm{hW}}+\Delta E_{\mathrm{hC}}=0 \\
& \quad \Delta E_{\mathrm{hW}}=-\Delta E_{\mathrm{hC}} \\
& m_{\mathrm{w}} \Delta T_{\mathrm{w}} c_{\mathrm{W}}=-m_{\mathrm{C}} \Delta T_{\mathrm{C}} c_{\mathrm{C}}, \text { where }\left(c_{\mathrm{C}}=c_{\mathrm{W}}\right) \\
& \Delta E_{\mathrm{hC}}+\Delta E_{\mathrm{hW}}=0 \\
& m_{\mathrm{C}} \Delta T_{\mathrm{C}} c_{\mathrm{C}}+m_{\mathrm{w}} \Delta T_{\mathrm{W}} c_{\mathrm{W}}=0, \text { where }\left(c_{\mathrm{C}}=c_{\mathrm{W}}\right) \\
& (29.0 \mathrm{~g})\left(T_{\mathrm{f}}-52.0^{\circ} \mathrm{C}\right) c=-(20.0 \mathrm{~g})\left(T_{\mathrm{f}}-10.0^{\circ} \mathrm{C}\right) c
\end{aligned}
$$

Divide both sides by $c$.

$$
\begin{gathered}
(29.0 \mathrm{~g})\left(T_{\mathrm{f}}-52.0^{\circ} \mathrm{C}\right)=-(20.0 \mathrm{~g})\left(T_{\mathrm{f}}-10.0^{\circ} \mathrm{C}\right) \\
(29.0) T_{\mathrm{f}}-1.508 \times 10^{3} \mathrm{~g} \cdot{ }^{\circ} \mathrm{C} \\
\quad=-(20.0 \mathrm{~g}) T_{\mathrm{f}}+200.0 \mathrm{~g} \cdot{ }^{\circ} \mathrm{C} \\
(29.0) T_{\mathrm{f}}+(20.0 \mathrm{~g}) T_{\mathrm{f}} \\
\quad=200.0 \mathrm{~g} \cdot{ }^{\circ} \mathrm{C}+1.508 \times 10^{3} \mathrm{~g} \cdot{ }^{\circ} \mathrm{C} \\
(49.0 \mathrm{~g}) T_{\mathrm{f}}= \\
1.708 \times 10^{3} \mathrm{~g} \cdot{ }^{\circ} \mathrm{C} \\
T_{\mathrm{f}}= \\
=\frac{1.708 \times 10^{3} \mathrm{~g} \cdot{ }^{\circ} \mathrm{C}}{49.0 \mathrm{~g}} \\
=34.9^{\circ} \mathrm{C}
\end{gathered}
$$

Note: You do not need to change the mass to kilograms because the mass unit cancels out in the end.

## Practice Test

## PRACTICE EXERCISES ANSWERS AND SOLUTIONS

1. When work is done in lifting an object, the gravitational potential energy is increased. When work is done on an object in accelerating it, the kinetic energy is changed. When work is done on an object to overcome friction, mechanical energy is changed to thermal energy.
2. When work is done in lifting an object, the gravitational potential energy is increased. When work is done on an object in accelerating from rest, the kinetic energy is increased.
3. $W=F d$

No work is done because there is no displacement.
4. Friction is a non-conservative force. This is because the amount of work done on the object does depend on the path. The longer the path length, the more work done on the object by friction.
5. If the velocity is constant, there is no net force. Therefore, the applied force is equal to the force of friction. Note that the force of friction is constant. This means that the applied force to overcome friction is also constant. The following graph shows a sketch of what this scenario should look like.

6. In dragging the object, work is done to overcome friction. When work is done to overcome friction, mechanical energy is changed into thermal energy.
7. Substitute $\frac{1}{2} v$ into the equation for kinetic energy to determine the effect of halving the velocity of an object in motion.

$$
\begin{aligned}
E_{\mathrm{k}} & =\frac{1}{2} m v^{2} \\
E_{\mathrm{k} 1 / 2} & =\frac{1}{2} m\left(\frac{1}{2} v\right)^{2} \\
& =\frac{1}{2} m\left(\frac{1}{4} v^{2}\right) \\
& =\frac{1}{4}\left(\frac{1}{2} m v^{2}\right) \\
& =\frac{1}{4} E_{\mathrm{k}}
\end{aligned}
$$

$\therefore$ When $v$ is halved, $E_{\mathrm{k}}$ is quartered.
8.


Work is represented algebraically by $W=F d$. The area under the curve on a force-displacement graph is equal to $F d$. Therefore, work is the area under the curve.
9. Gravity is a conservative force. Spring forces are conservative forces. These are conservative because the amount of work done is independent of the path taken. Mechanical energy is conserved.
10. a) The gravitational potential energy does not depend on the velocity. Therefore, the gravitational potential energy if Elissa ran up the stairs at $2 v$ instead would still be $\mathrm{E}_{\mathrm{p}}$.
b) The power used by Elissa to run up the stairs is represented by the equation $P=F v$.
Substitute $2 v$ into the equation to determine the effect of doubling the velocity.
$P_{1}=F v$
$P_{2}=F(2 v)$
$P_{2}=2 F v$
$P_{2}=2 P_{1}$
Therefore, if the velocity doubles, the power doubles. This can be represented by the expression $2 P$.
11. $P=F v$

$$
\begin{aligned}
& =m g v \\
& =(45 \mathrm{~kg})\left(9.81 \mathrm{~m} / \mathrm{s}^{2}\right)(0.25 \mathrm{~m} / \mathrm{s}) \\
& =110 \mathrm{~W}
\end{aligned}
$$

Lifting a 45 kg student vertically at a speed of $0.25 \mathrm{~m} / \mathrm{s}$ will require 110 W .
12. Aluminum has the greatest specific heat capacity. It requires more heat to raise its temperature. Therefore, it could remove more heat from the liquid than the other metals.
13. $\Delta E_{\mathrm{h}}=m \Delta T c$

The temperature change in metal bar 2 is greater than the change in metal bar 1 . This means that metal bar 2 has a smaller specific heat capacity then metal bar 1. It takes less energy to raise the substance's temperature, but because the same amount of heat was applied to each bar, metal bar 2 reaches a higher temperature.

$$
\text { 14. } \begin{aligned}
\Delta E_{\mathrm{hX}}+\Delta E_{\mathrm{hY}} & =0 \\
m_{\mathrm{X}} \Delta T_{\mathrm{X}} c_{\mathrm{X}} & =-m_{\mathrm{Y}} \Delta T_{\mathrm{Y}} c_{\mathrm{Y}} \\
(1 \mathrm{~kg})\left(62^{\circ} \mathrm{C}-86^{\circ} \mathrm{C}\right)\left(c_{\mathrm{X}}\right) & =-(2 \mathrm{~kg})\left(62^{\circ} \mathrm{C}-24^{\circ} \mathrm{C}\right)\left(c_{\mathrm{Y}}\right) \\
-24 c_{\mathrm{X}} & =-76 c_{\mathrm{Y}} \\
c_{\mathrm{X}} & =3.2 c_{\mathrm{Y}}
\end{aligned}
$$

Therefore, liquid X has the higher specific heat capacity. Its specific heat capacity is 3.2 times greater than the specific heat capacity of liquid Y.
15. $\Delta E_{\mathrm{hX}}+\Delta E_{\mathrm{hY}}=0$

$$
\begin{aligned}
m_{\mathrm{X}} \Delta T_{\mathrm{X}} c_{\mathrm{X}} & =-m_{\mathrm{Y}} \Delta T_{\mathrm{Y}} c_{\mathrm{Y}} \\
(1 \mathrm{~kg})\left(34^{\circ} \mathrm{C}-10^{\circ} \mathrm{C}\right)\left(c_{\mathrm{X}}\right) & =-(2 \mathrm{~kg})\left(34^{\circ} \mathrm{C}-58^{\circ} \mathrm{C}\right)\left(c_{\mathrm{Y}}\right) \\
24 c_{\mathrm{X}} & =48 c_{\mathrm{Y}} \\
c_{\mathrm{X}} & =2.0 c_{\mathrm{Y}}
\end{aligned}
$$

Therefore, liquid X has the higher specific heat capacity. Its specific heat capacity is 2.0 times greater than the specific heat capacity of liquid Y.
16. $\Delta E_{\mathrm{k}}+\Delta E_{\mathrm{p}}=0$ or $\Delta E_{\mathrm{k}}=-\Delta E_{\mathrm{p}}$

As the object reaches its maximum height, the kinetic energy will decrease (the object slows) while the potential energy will increase (the object's height increases). The following sketch shows this relationship in graph form.

17. $E_{\mathrm{k}}=\frac{1}{2} m v^{2}$ or $E_{\mathrm{k}} \propto v^{2}$

Therefore, as the velocity increases, the kinetic energy will also increase but at an increasing rate. The following sketch illustrates this exponential growth in graph form.

18. A machine is never $100 \%$ efficient because some of the mechanical energy used by the machine changes into heat energy due to frictional forces. This heat energy is waste that lowers the efficiency of the machine.
19. $V=I R$

$$
\begin{aligned}
& =(9.0 \mathrm{~A})(5.0 \Omega) \\
& =45 \mathrm{~V}
\end{aligned}
$$

The potential difference across a $5.0 \Omega$ resistor is 45 V .
20. $I=\frac{q}{t}$
$q=I t$

$$
=(0.20 \mathrm{~A})(60.0 \mathrm{~s})
$$

$$
=12.0 \mathrm{C}
$$

$\frac{1 \mathrm{e}^{-}}{1.60 \times 10^{-19} \mathrm{C}}=\frac{\# \mathrm{e}^{-}}{12.0 \mathrm{C}}$

$$
\# \mathrm{e}^{-}=7.5 \times 10^{19}
$$

A conductor carrying a current of 0.20 A will have $7.5 \times 10^{19}$ electrons pass through it each minute.
21. $P=\frac{\Delta E}{t}$

$$
\begin{aligned}
\Delta E & =P t \\
& =(775 \mathrm{~W})(60 \mathrm{~s} / \mathrm{min})(15 \mathrm{~min}) \\
& =7.0 \times 10^{5} \mathrm{~J}
\end{aligned}
$$

A 775 W heating coil will use $7.0 \times 10^{5} \mathrm{~J}$ of energy when it works for 15 minutes.
22. Add the parallel resistors first:

$$
\begin{aligned}
\frac{1}{R_{\mathrm{eq}(\mathrm{p})}} & =\frac{1}{R_{1}}+\frac{1}{R_{2}} \\
\frac{1}{R_{\mathrm{eq}(\mathrm{p})}} & =\frac{1}{5.0 \Omega}+\frac{1}{5.0 \Omega} \\
\frac{1}{R_{\mathrm{eq}(\mathrm{p})}} & =\frac{2}{5.0 \Omega} \\
R_{\mathrm{eq}(\mathrm{p})} & =\frac{5.0 \Omega}{2} \\
R_{\mathrm{eq}(\mathrm{p})} & =2.5 \Omega
\end{aligned}
$$

Now add in the series resistor:

$$
\begin{aligned}
R_{\mathrm{eq}(\mathrm{~s})} & =2.5 \Omega+5.0 \Omega \\
& =7.5 \Omega
\end{aligned}
$$

The equivalent resistance of the three resistors is $7.5 \Omega$.
23. $R_{\mathrm{eq}}=R_{1}+R_{2}$

$$
\begin{aligned}
& =5.00 \Omega+7.50 \Omega \\
& =12.5 \Omega
\end{aligned}
$$

The equivalent resistance of the two resistors is $12.5 \Omega$.
24. In order to determine the resistance through an electric circuit, you would use an ammeter to measure the current through the circuit and a voltmeter to measure the drop in potential across the resistor. From this information, you could determine the resistance in the circuit.
25. For every electron that passes through $R_{2}$, two electrons will pass through $R_{3}\left(R_{3}\right.$ has $\frac{1}{2}$ the resistance). This means that for every three electrons that pass through this circuit, one electron passes through $R_{2}$ (i.e., $\frac{1}{3}$ of the total current passes through $R_{2}$ ).

$$
\begin{aligned}
I_{\mathrm{R}_{2}} & =\frac{1}{3} \times I_{\mathrm{R}_{2}} \\
& =\frac{1}{3} \times 9.0 \mathrm{~A} \\
& =3.0 \mathrm{~A}
\end{aligned}
$$

The current through $R_{2}$ is 3.0 A .
26. The voltage across each resistor must be the same (Kirchhoff's laws). Therefore, when two resistors are connected in parallel in an electric circuit, they will both have the same voltage drop.
27. $V=I R \quad \therefore R=\frac{V}{I}$

If two copper conductors have the same resistance, they must have the same voltage-to-current ratio.
28. By adding $R_{2}$ in series, the resistance in the circuit increases, $R_{\text {eq }}=R_{1}+R_{2}$. When the resistance of the circuit increases, the current through the circuit decreases.
$I=\frac{V}{R}$
29. By adding $R_{2}$ in parallel, the resistance in the circuit decreases, but the resistance of $R_{1}$ does not change. The voltage across each resistor in a parallel circuit also does not change by adding $R_{2}$.

$$
P=\frac{V^{2}}{R}
$$

Therefore, the power dissipated by $R_{1}$ also does not change when resistor $R_{2}$ is added in parallel.
30. a) Find the equivalent resistance of the parallel resistors by adding their resistances:

$$
\begin{aligned}
\frac{1}{R_{\mathrm{eq}(\mathrm{p})}} & =\frac{1}{R_{2}}+\frac{1}{R_{3}} \\
\frac{1}{R_{\mathrm{eq}(\mathrm{p})}} & =\frac{1}{4.0 \Omega}+\frac{1}{2.0 \Omega} \\
\frac{1}{R_{\mathrm{eq}(\mathrm{p})}} & =\frac{1}{4.0 \Omega}+\frac{2}{4.0 \Omega} \\
\frac{1}{R_{\mathrm{eq}(\mathrm{p})}} & =\frac{3}{4.0 \Omega} \\
R_{\mathrm{eq}(\mathrm{p})} & =\frac{4.0 \Omega}{3} \\
R_{\mathrm{eq}(\mathrm{p})} & =1.33 \Omega
\end{aligned}
$$

Now find the total equivalent resistance in the circuit by adding the resistor in series:

$$
\begin{aligned}
R_{\mathrm{eq}(\mathrm{~s})} & =1.33 \Omega+R_{1} \\
& =1.33 \Omega+5.0 \Omega \\
& =6.33 \Omega \\
& \doteq 6.3 \Omega
\end{aligned}
$$

Therefore, the total resistance of the circuit is $6.3 \Omega$.
b) $\quad V=I R$

$$
\begin{aligned}
& =(1.6 \mathrm{~A})(6.3 \Omega) \\
& =10 \mathrm{~V}
\end{aligned}
$$

The terminal voltage of the battery is 10 V .

## WAVES AND SOUND

## Lesson 1—Waves and Sound

PRACTICE EXERCISES ANSWERS AND SOLUTIONS

1. $v=\lambda f$
$\lambda=\frac{v}{f}$

$$
\begin{aligned}
& =\frac{3.3 \times 10^{-1} \mathrm{~m} / \mathrm{s}}{5.0 \times 10^{-1} \mathrm{~Hz}} \\
& =0.66 \mathrm{~m}
\end{aligned}
$$

2. $v=\frac{d}{t}$

$$
\begin{aligned}
& =\frac{4.60 \mathrm{~m}}{2.00 \mathrm{~s}} \\
& =2.30 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

3. $v=\lambda f$

$$
\begin{aligned}
f & =\frac{2.5 \mathrm{~m} / \mathrm{s}}{5.0 \mathrm{~m}} \\
& =0.500 \mathrm{~Hz} \\
T & =\frac{1}{f} \\
& =\frac{1}{0.500 \mathrm{~Hz}} \\
& =2.0 \mathrm{~s}
\end{aligned}
$$

4. Frequency is the number of waves per second.

$$
\begin{aligned}
\frac{9.5}{60.0 \mathrm{~s}} & =f \\
f & =0.16 \mathrm{~Hz}
\end{aligned}
$$

5. $f=\frac{1}{T}$

$$
\begin{aligned}
& =\frac{1}{1.00 \times 10^{-2} \mathrm{~Hz}} \\
& =1.00 \times 10^{2} \mathrm{~Hz} \\
v & =\lambda f \\
\lambda & =\frac{v}{f} \\
& =\frac{335 \mathrm{~m} / \mathrm{s}}{1.00 \times 10^{2} \mathrm{~Hz}} \\
& =3.35 \mathrm{~m}
\end{aligned}
$$

6. $v=\lambda f$

$$
\begin{aligned}
\lambda & =\frac{v}{f} \\
& =\frac{3.00 \times 10^{8} \mathrm{~m} / \mathrm{s}}{1.00 \times 10^{6} \mathrm{~Hz}} \\
& =3.00 \times 10^{2} \mathrm{~m}
\end{aligned}
$$

7. $v=\lambda f$

$$
\begin{aligned}
f & =\frac{v}{\lambda} \\
& =\frac{3.00 \times 10^{8} \mathrm{~m} / \mathrm{s}}{5.00 \times 10^{-7} \mathrm{~m}} \\
& =6.000 \times 10^{14} \mathrm{~Hz} \\
T & =\frac{1}{f} \\
& =\frac{1}{6.000 \times 10^{14} \mathrm{~Hz}} \\
& =1.67 \times 10^{-15} \mathrm{~s}
\end{aligned}
$$

## Lesson 2—Properties of Waves

PRACTICE EXERCISES ANSWERS AND SOLUTIONS

1. $2.3 \mathrm{~cm}+1.9 \mathrm{~cm}=4.2 \mathrm{~cm}$
2. $5.2 \mathrm{~cm}+2.5 \mathrm{~cm}=7.7 \mathrm{~cm}$
3. $2.3 \mathrm{~cm}-1.9 \mathrm{~cm}=0.4 \mathrm{~cm}$
4. $5.2 \mathrm{~cm}-2.5 \mathrm{~cm}=2.7 \mathrm{~cm}$
5. 


6.

7.


## Lesson 3-Sound

## PRACTICE EXERCISES ANSWERS AND SOLUTIONS

1. $v=\left(331+\left(0.59 /{ }^{\circ} \mathrm{C}\right) T\right) \mathrm{m} / \mathrm{s}$

$$
\begin{aligned}
& =\left(331+\left(0.59 /{ }^{\circ} \mathrm{C}\right)\left(10.0^{\circ} \mathrm{C}\right)\right) \mathrm{m} / \mathrm{s} \\
& =337 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

2. $\#$ beats $=f_{1}-f_{2}$

$$
\begin{aligned}
& =256 \mathrm{~Hz}-251 \mathrm{~Hz} \\
& =5 \mathrm{~Hz}
\end{aligned}
$$

3. $v=\frac{d}{t}$

$$
\begin{aligned}
d & =v t \\
& =(341 \mathrm{~m} / \mathrm{s})(5.50 \mathrm{~s}) \\
& =1.88 \times 10^{3} \mathrm{~m}
\end{aligned}
$$

4. $v=\lambda f$

$$
\begin{aligned}
\lambda & =\frac{v}{f} \\
& =\frac{343 \mathrm{~m} / \mathrm{s}}{27.5 \mathrm{~Hz}} \\
& =12.5 \mathrm{~m}
\end{aligned}
$$

5. $\quad v=\left(331+\left(0.59 /{ }^{\circ} \mathrm{C}\right) T\right) \mathrm{m} / \mathrm{s}$

$$
\begin{aligned}
& =\left(331+\left(0.59 /{ }^{\circ} \mathrm{C}\right)\left(22^{\circ} \mathrm{C}\right)\right) \mathrm{m} / \mathrm{s} \\
& =344.0 \mathrm{~m} / \mathrm{s} \\
v & =\frac{d}{t} \\
d & =v t \\
& =(344.0 \mathrm{~m} / \mathrm{s})(3.6 \mathrm{~s}) \\
& =1.2 \times 10^{3} \mathrm{~m}
\end{aligned}
$$

6. $v=\frac{d}{t}$

$$
\begin{aligned}
t & =\frac{1}{2} t_{\text {total }} \\
& =0.310 \mathrm{~s} \\
d & =v t \\
& =\left(1.46 \times 10^{3} \mathrm{~m} / \mathrm{s}\right)(0.310 \mathrm{~s}) \\
& =4.5 \times 10^{2} \mathrm{~m}
\end{aligned}
$$

7. $v=\left(331+\left(0.59 /{ }^{\circ} \mathrm{C}\right) T\right) \mathrm{m} / \mathrm{s}$

$$
=\left(331+\left(0.59 /{ }^{\circ} \mathrm{C}\right)\left(18.0^{\circ} \mathrm{C}\right)\right) \mathrm{m} / \mathrm{s}
$$

$$
=342.0 \mathrm{~m} / \mathrm{s}
$$

$$
v=\frac{d}{t}
$$

$$
t=\frac{d}{v}
$$

$$
=\frac{1.0 \times 10^{4} \mathrm{~m}}{342.0 \mathrm{~m} / \mathrm{s}}
$$

$$
=29.2 \mathrm{~s}
$$

8. $v=\left(331+\left(0.59 /{ }^{\circ} \mathrm{C}\right) T\right) \mathrm{m} / \mathrm{s}$

$$
\begin{aligned}
& =\left(331+\left(0.59 /{ }^{\circ} \mathrm{C}\right)\left(20.0^{\circ} \mathrm{C}\right)\right) \mathrm{m} / \mathrm{s} \\
& =343 \mathrm{~m} / \mathrm{s} \\
v & =\lambda f \\
\lambda & =\frac{v}{f} \\
& =\frac{343.0 \mathrm{~m} / \mathrm{s}}{2.50 \times 10^{5} \mathrm{~Hz}} \\
& =1.37 \times 10^{-3} \mathrm{~m}
\end{aligned}
$$

9. $\quad v=\left(331+\left(0.59 /{ }^{\circ} \mathrm{C}\right) T\right) \mathrm{m} / \mathrm{s}$

$$
355=\left(331+\left(0.59 /{ }^{\circ} \mathrm{C}\right) T\right) \mathrm{m} / \mathrm{s}
$$

$$
\begin{aligned}
T & =\left(\frac{355-331}{0.59}\right){ }^{\circ} \mathrm{C} \\
& =40.7^{\circ} \mathrm{C}
\end{aligned}
$$

$$
=40.7^{\circ} \mathrm{C}
$$

10. $\#$ beats $=f_{1}-f_{2}$

$$
\begin{aligned}
f_{2} & =6.60 \times 10^{2} \mathrm{~Hz} \pm 3.00 \mathrm{~Hz} \\
& =6.63 \times 10^{2} \mathrm{~Hz} \text { or } 6.57 \times 10^{2} \mathrm{~Hz}
\end{aligned}
$$

11. $v=\lambda f$

$$
\begin{aligned}
& =(1.5 \mathrm{~m})\left(1.00 \times 10^{3} \mathrm{~Hz}\right) \\
& =1.5 \times 10^{3} \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

12. $v=\left(331+\left(0.59 /{ }^{\circ} \mathrm{C}\right) T\right) \mathrm{m} / \mathrm{s}$
$=\left(331+\left(0.59 /{ }^{\circ} \mathrm{C}\right)\left(17.0^{\circ} \mathrm{C}\right)\right) \mathrm{m} / \mathrm{s}$

$$
=341.0 \mathrm{~m} / \mathrm{s}
$$

$v=\frac{d}{t}$

$$
d=v t
$$

$$
=(341.0 \mathrm{~m} / \mathrm{s})(1.05 \mathrm{~s})
$$

$$
=358.1 \mathrm{~m}
$$

$$
\begin{aligned}
& t=\frac{1}{2} t_{\text {total }} \\
& \therefore d=\frac{1}{2} d_{\text {total }} \\
& d=\frac{358.1 \mathrm{~m}}{2} \\
&=179 \mathrm{~m}
\end{aligned}
$$

13. $T=\frac{1}{f}$

$$
\begin{aligned}
& =\frac{1}{256 \mathrm{~Hz}} \\
& =3.91 \times 10^{-3} \mathrm{~s}
\end{aligned}
$$

14. $f=\frac{1}{T}$

$$
\begin{aligned}
& =\frac{1}{2.0 \times 10^{-5} \mathrm{~s}} \\
& =5.00 \times 10^{4} \mathrm{~Hz} \\
v & =\lambda f \\
& =\left(3.0 \times 10^{-2} \mathrm{~m}\right)\left(5.00 \times 10^{4} \mathrm{~Hz}\right) \\
& =1.5 \times 10^{3} \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

## Lesson 4-Characteristics of Sound

## PRACTICE EXERCISES

 ANSWERS AND SOLUTIONS1. $\lambda=2 L$

$$
\begin{aligned}
& =2(1.2 \mathrm{~m}) \\
& =2.40 \mathrm{~m} \\
v & =\lambda f \\
& =(2.40 \mathrm{~m})(65 \mathrm{~Hz}) \\
& =1.6 \times 10^{2} \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

2. $f=\frac{144 \mathrm{~Hz}}{3}$

$$
=48.0 \mathrm{~Hz}
$$

3. Third harmonic $=3$ (Fundamental frequency)

$$
\begin{aligned}
& =3(354 \mathrm{~Hz}) \\
& =1.06 \times 10^{3} \mathrm{~Hz}
\end{aligned}
$$

4. $f=\frac{2.5 \times 10^{3} \mathrm{~Hz}}{4}$

$$
=6.3 \times 10^{2} \mathrm{~Hz}
$$

5. Third harmonic $=3$ (Fundamental frequency)

$$
\begin{aligned}
\text { Fundamental Frequency } & =\frac{3 \mathrm{rd} \text { harmonic }}{3} \\
& =\frac{1.2 \times 10^{3} \mathrm{~Hz}}{3} \\
& =4.00 \times 10^{2} \mathrm{~Hz}
\end{aligned}
$$

2 nd harmonic $=2($ Fundamental frequency $)$

$$
\begin{aligned}
& =2\left(4.00 \times 10^{2} \mathrm{~Hz}\right) \\
& =8.0 \times 10^{2} \mathrm{~Hz}
\end{aligned}
$$

6. Fundamental frequency $=\frac{335 \mathrm{~Hz}}{3}$

$$
=111.7 \mathrm{~Hz}
$$

4th harmonic $=4($ Fundamental frequency $)$

$$
\begin{aligned}
& =4(111.7 \mathrm{~Hz}) \\
& =447 \mathrm{~Hz}
\end{aligned}
$$

7. $\lambda=2 L$

$$
\begin{aligned}
& =2(27.0 \mathrm{~cm}) \\
& =54.00 \mathrm{~cm} \\
v & =\lambda f \\
& =(54.00 \mathrm{~cm})(637 \mathrm{~Hz}) \\
& =3.440 \times 10^{4} \mathrm{~cm} / \mathrm{s}
\end{aligned}
$$

Frequency of 22.0 cm string

$$
\begin{aligned}
\lambda & =2 L \\
& =2(22.0 \mathrm{~cm}) \\
& =44.00 \mathrm{~cm} \\
f & =\frac{v}{\lambda} \\
& =\frac{3.440 \times 10^{4} \mathrm{~cm} / \mathrm{s}}{44.00 \mathrm{~cm}} \\
& =782 \mathrm{~Hz}
\end{aligned}
$$

8. $\lambda=2 L$

$$
\begin{aligned}
& =2(25 \mathrm{~cm}) \\
& =50.0 \mathrm{~cm} \\
v & =\lambda f \\
& =(50.0 \mathrm{~cm})(441 \mathrm{~Hz}) \\
& =2.205 \times 10^{4} \mathrm{~cm} / \mathrm{s}
\end{aligned}
$$

Length of 525 Hz string

$$
\begin{aligned}
v & =\lambda f \\
\lambda & =\frac{v}{f} \\
& =\frac{2.205 \times 10^{4} \mathrm{~cm} / \mathrm{s}}{525 \mathrm{~Hz}} \\
& =42.00 \mathrm{~cm}
\end{aligned}
$$

$$
\begin{aligned}
\lambda & =2 L \\
L & =\frac{\lambda}{2} \\
& =\frac{42.00 \mathrm{~cm}}{2} \\
& =21 \mathrm{~cm}
\end{aligned}
$$

9. $v=\sqrt{\frac{T}{m / L}}$

$$
\begin{aligned}
& =\sqrt{\frac{23 \mathrm{~N}}{1.9 \times 10^{-3} \mathrm{~kg} / 0.80 \mathrm{~m}}} \\
& =98 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

10. $\lambda=2 L$

$$
\begin{aligned}
& =2(0.80 \mathrm{~m}) \\
& =1.60 \mathrm{~m} \\
v & =\lambda f \\
f & =\frac{v}{\lambda} \\
& =\frac{98 \mathrm{~m} / \mathrm{s}}{1.60 \mathrm{~m}} \\
& =62 \mathrm{~Hz}
\end{aligned}
$$

11. $v=\sqrt{\frac{T}{m / L}}$

$$
=\sqrt{\frac{80.0 \mathrm{~N}}{195 \times 10^{-3} \mathrm{~kg} / 5.0 \mathrm{~m}}}
$$

$$
=45.3 \mathrm{~m} / \mathrm{s}
$$

$$
\lambda=2 L
$$

$$
=2(5.0 \mathrm{~m})
$$

$$
=10.0 \mathrm{~m}
$$

$$
v=\lambda f
$$

$$
f=\frac{45.3 \mathrm{~m} / \mathrm{s}}{10.0 \mathrm{~m}}
$$

$$
=4.5 \mathrm{~Hz}
$$

12. $\lambda=2 L$

$$
\begin{aligned}
& =2(2.6 \mathrm{~m}) \\
& =5.20 \mathrm{~m} \\
v & =\lambda f \\
& =(5.20 \mathrm{~m})(133 \mathrm{~Hz}) \\
& =691.6 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

$$
\begin{aligned}
v & =\sqrt{\frac{T}{m / L}} \\
691.6 \mathrm{~m} / \mathrm{s} & =\sqrt{\frac{T}{9.8 \times 10^{-2} \mathrm{~kg} / 2.6 \mathrm{~m}}} \\
T & =(691.6 \mathrm{~m} / \mathrm{s})^{2}\left(\frac{9.8 \times 10^{-2} \mathrm{~kg}}{2.6 \mathrm{~m}}\right) \\
& =1.8 \times 10^{4} \mathrm{~N}
\end{aligned}
$$

13. $v=\lambda f$

$$
\begin{aligned}
& \quad v=\sqrt{\frac{T}{m / L}} \\
& \therefore \lambda f=\sqrt{\frac{T}{m / L}} \\
& \text { or } f \alpha \sqrt{T} \\
& \text { or } T \alpha f^{2}
\end{aligned}
$$

Frequency

$$
\begin{aligned}
\frac{\mathrm{To}}{\text { From }} & =\frac{231 \mathrm{~Hz}}{224 \mathrm{~Hz}} \\
& =1.031 \\
f^{2} & =1.0635 \\
T & =\left(1.00 \times 10^{2} \mathrm{~N}\right)(1.0635) \\
& =1.06 \times 10^{2} \mathrm{~N}
\end{aligned}
$$

14. $v=\sqrt{\frac{T}{m / L}}$

$$
\begin{aligned}
65 \mathrm{~m} / \mathrm{s} & =\sqrt{\frac{12 \mathrm{~N}}{m / 0.50 \mathrm{~m}}} \\
m & =\frac{(12 \mathrm{~N})(0.50 \mathrm{~m})}{(65 \mathrm{~m} / \mathrm{s})^{2}} \\
& =1.4 \times 10^{-3} \mathrm{~kg}
\end{aligned}
$$

## Lesson 5-Air Columns

## PRACTICE EXERCISES ANSWERS AND SOLUTIONS

1. $v=\lambda f$

$$
\begin{aligned}
\lambda & =\frac{v}{f} \\
& =\frac{343 \mathrm{~m} / \mathrm{s}}{256 \mathrm{~Hz}} \\
& =1.34 \mathrm{~m}
\end{aligned}
$$

2. $\lambda=4 L$

$$
\begin{aligned}
& =4\left(1.89 \times 10^{-1} \mathrm{~m}\right) \\
& =7.560 \times 10^{-1} \mathrm{~m} \\
v & =\lambda f \\
& =\left(7.560 \times 10^{-1} \mathrm{~m}\right)\left(4.40 \times 10^{2} \mathrm{~Hz}\right) \\
& =333 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

3. 3 rd harmonic $=3$ (Fundamental Frequency)

$$
\begin{aligned}
& =3(384 \mathrm{~Hz}) \\
& =1.15 \times 10^{3} \mathrm{~Hz}
\end{aligned}
$$

4. $\lambda=4 L$

$$
\begin{aligned}
& =4\left(22.0 \times 10^{-2} \mathrm{~m}\right) \\
& =88.00 \times 10^{-2} \mathrm{~m}
\end{aligned}
$$

$$
v=\left(331+\left(0.59 /{ }^{\circ} \mathrm{C}\right) T\right) \mathrm{m} / \mathrm{s}
$$

$$
=\left(331+\left(0.59 /{ }^{\circ} \mathrm{C}\right)\left(20.0^{\circ} \mathrm{C}\right)\right) \mathrm{m} / \mathrm{s}
$$

$$
=342.8 \mathrm{~m} / \mathrm{s}
$$

$$
v=\lambda f
$$

$$
f=\frac{v}{\lambda}
$$

$$
=\frac{342.8 \mathrm{~m} / \mathrm{s}}{88.00 \times 10^{-2} \mathrm{~m}}
$$

$$
=3.90 \times 10^{2} \mathrm{~Hz}
$$

5. $v=\left(331+\left(0.59 /{ }^{\circ} \mathrm{C}\right) T\right) \mathrm{m} / \mathrm{s}$

$$
=\left(331+\left(0.59 /{ }^{\circ} \mathrm{C}\right)\left(20.0^{\circ} \mathrm{C}\right)\right) \mathrm{m} / \mathrm{s}
$$

$$
=342.8 \mathrm{~m} / \mathrm{s}
$$

$$
v=\lambda f
$$

$$
\lambda=\frac{v}{f}
$$

$$
=\frac{342.8 \mathrm{~m} / \mathrm{ss}}{256 \mathrm{~Hz}}
$$

$$
=1.339 \mathrm{~m}
$$

$$
\begin{aligned}
L & =\frac{\lambda}{4} \\
& =\frac{1.339 \mathrm{~m}}{4} \\
& =0.335 \mathrm{~m}
\end{aligned}
$$

6. $\lambda=4 L$

$$
\begin{aligned}
& =4(0.55 \mathrm{~m}) \\
& =2.20 \mathrm{~m} \\
v & =\lambda f \\
& =(2.20 \mathrm{~m})(156 \mathrm{~Hz}) \\
& =3.4 \times 10^{2} \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

7. $v=\lambda f$

$$
\lambda=\frac{v}{f}
$$

$$
=\frac{341 \mathrm{~m} / \mathrm{s}}{4.40 \times 10^{2} \mathrm{~Hz}}
$$

$$
=0.7750 \mathrm{~m}
$$

$$
L=\frac{\lambda}{4}
$$

$$
=\frac{0.7750 \mathrm{~m}}{4}
$$

$$
=0.194 \mathrm{~m}
$$

8. $v=\lambda f$
$\lambda=\frac{v}{f}$
$=\frac{341 \mathrm{~m} / \mathrm{s}}{256 \mathrm{~Hz}}$
$=1.332 \mathrm{~m}$
$L=\frac{\lambda}{2}$
$=\frac{1.332 \mathrm{~m}}{2}$
$=0.666 \mathrm{~m}$
9. $\lambda=2 L$

$$
\begin{aligned}
& =2(0.330 \mathrm{~m}) \\
& =0.6600 \mathrm{~m} \\
v & =\lambda f \\
& =(0.6600 \mathrm{~m})(512 \mathrm{~Hz}) \\
& =338 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

10. 2nd harmonic

2 nd harmonic $=2($ Fundamental $)$

$$
\begin{aligned}
& =2(384 \mathrm{~Hz}) \\
& =768 \mathrm{~Hz}
\end{aligned}
$$

11. 3rd harmonic

3 rd harmonic $=3($ Fundamental $)$

$$
\begin{aligned}
& =3(384 \mathrm{~Hz}) \\
& =1.15 \times 10^{3} \mathrm{~Hz}
\end{aligned}
$$

## Lesson 6-Doppler Effect

## PRACTICE EXERCISES ANSWERS AND SOLUTIONS

1. Sound waves are being emitted from the train's whistle in all directions. The car travels toward these expanding wavefronts and the driver hears an apparent higher pitch than the source.
2. The car is now travelling away from the expanding wavefronts. Thus, the driver of the car hears an apparent lower pitch than the source.
3. As the whistle moves toward the stationary observer, the apparent wavelength of the whistle wavefronts decreases (i.e., the wavefronts become compressed in front of the whistle). Thus, the observer hears an apparently higher frequency (pitch) sound than the source.
4. As the whistle moves toward the stationary observer, the apparent wavelength of the whistle wavefronts increases (i.e., the wavefronts become farther away from each other behind the whistle). Thus, the observer hears an apparently lower frequency (pitch) sound than the source.

## Practice Test

## ANSWERS AND SOLUTIONS

1. The apparent change in frequency due to the motion of the source or the observer is called the Doppler effect.
$\mathbf{C}$ is the correct answer.
2. $v=\lambda f$

$$
\begin{aligned}
f & =\frac{v}{\lambda} \\
& =\frac{10.0 \mathrm{~cm} / \mathrm{s}}{2.0 \mathrm{~cm}} \\
& =5.0 \mathrm{~Hz}
\end{aligned}
$$

3. When waves meet crest to crest or trough to trough, they are meeting in phase. When waves meet in phase, constructive interference results.
$\mathbf{A}$ is the correct answer.
4. 



When the sound source is approaching the observer, he or she is in the region of high frequency.

In the same way, if the sound source is moving away from the observer, he is in the region of low frequency. The region of lowest frequency is represented by position A .
$\mathbf{A}$ is the correct answer.
5. A wavelength is the distance between two adjacent points that are in phase. Points A and E are two such points.
$\mathbf{C}$ is the correct answer.
6. A particle in a transverse wave speeds up and slows down. The point reverses its direction at A ; therefore, as the particle approaches point A , it is slowing down. Conversely, as the particle approaches the equilibrium position (B) it is speeding up.

B is the correct answer.
7. Each segment (loop) represents $\frac{1}{2} \lambda$. Therefore,

$$
\begin{aligned}
\frac{3}{2} \lambda & \lambda=4.5 \mathrm{~m} \\
\lambda & =3.0 \mathrm{~m} \\
v & =\lambda f \\
& =(3.0 \mathrm{~m})(60.0 \mathrm{~Hz}) \\
& =1.8 \times 10^{2} \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

8. The Doppler effect occurs when there is relative motion between a wave source and the observer.

B is the correct answer.
9. Resonance results when two objects have the same natural frequency, and the vibration of one of the objects causes the other object to vibrate.

D is the correct answer.
10. Diffraction, constructive interference, and destructive interference are all exclusive properties of waves. Due to the presence of various fields and forces, particles can be refracted.
$\mathbf{A}$ is the correct answer.
11. The amplitude is the maximum displacement from the equilibrium position. The amplitude is 10 cm .
$\mathbf{C}$ is the correct answer.
12. $v=f \lambda$

$$
\begin{aligned}
& =\left(1.02 \times 10^{4} \mathrm{~Hz}\right)(0.491 \mathrm{~m}) \\
& =5.008 \times 10^{3} \mathrm{~m} / \mathrm{s} \\
d & =v t \Rightarrow t=\frac{d}{v} \\
& =\frac{13.0 \mathrm{~m}}{5.008 \times 10^{3} \mathrm{~m} / \mathrm{s}} \\
& =2.60 \times 10^{-3} \mathrm{~s} \\
& =2.60 \mathrm{~ms}
\end{aligned}
$$

13. 



A is the correct answer.
14. Determine the frequency, as it does not vary with the medium the light propagates through.

$$
\begin{aligned}
v & =f \lambda \Rightarrow f=\frac{v}{\lambda} \\
& =\frac{3.00 \times 10^{8} \mathrm{~m} / \mathrm{s}}{7.0 \times 10^{-7} \mathrm{~m}} \\
& =4.29 \times 10^{14} \mathrm{~Hz}
\end{aligned}
$$

Find the wavelength in glass.

$$
\begin{aligned}
v & =f \lambda \Rightarrow \lambda=\frac{v}{f} \\
& =\frac{2.0 \times 10^{8} \mathrm{~m} / \mathrm{s}}{4.29 \times 10^{14} \mathrm{~Hz}} \\
& =4.7 \times 10^{-7} \mathrm{~m}
\end{aligned}
$$

15. $f_{\mathrm{b}}=f_{2}-f_{1}$

$$
\begin{aligned}
& =440 \mathrm{~Hz}-435 \mathrm{~Hz} \\
& =5.0 \mathrm{~Hz}
\end{aligned}
$$

The number of beats heard in 5.0 s can be calculated.

$$
(5.0 \mathrm{~Hz})(5.0 \mathrm{~s})=25 \text { beats }
$$

16. $f_{\mathrm{b}}=f_{2}-f_{1}$

$$
\begin{aligned}
f_{\mathrm{b}} & =\frac{20.0}{8.0}=2.50 \mathrm{~Hz} \\
2.50 \mathrm{~Hz} & =525 \mathrm{~Hz}-f_{1} \\
f_{1} & =522.5 \mathrm{~Hz}
\end{aligned}
$$

$$
\begin{aligned}
v & =f \lambda \\
\lambda & =\frac{v}{f} \\
& =\frac{342 \mathrm{~m} / \mathrm{s}}{522.5 \mathrm{~Hz}} \\
& =0.65 \mathrm{~m}
\end{aligned}
$$

17. A pure note of a given frequency has no overtones and vibrates only at the fundamental frequency. Sound coming from two different musical instruments will be recognizably different, even when the same note is played, because additional frequencies, called overtones, are produced.
Thus, the quality of the sound produced by each instrument is different, allowing the instruments to be distinguished from one another.
An oscilloscope display of these two sounds would be very different because the display shows the superposition of these harmonics.
$\mathbf{C}$ is the correct answer.
18. Infrasonic waves are waves below the audible frequency range, which varies from one person to another. The typical audible frequency range is 20 Hz to 20 kHz . Therefore, the frequency 12 Hz is infrasonic.

A is the correct answer.
19. First, express the time taken by the projectile and the sound in terms of their respective distances and speeds.
$d_{\mathrm{p}}=v_{\mathrm{p}} t_{\mathrm{p}} \Rightarrow t_{\mathrm{p}}=\frac{d_{\mathrm{p}}}{v_{\mathrm{p}}}$
$d_{\mathrm{s}}=v_{\mathrm{s}} t_{\mathrm{s}} \Rightarrow t_{\mathrm{s}}=\frac{d_{\mathrm{s}}}{v_{\mathrm{s}}}$
The difference in time is known.
$\frac{d_{\mathrm{s}}}{v_{\mathrm{s}}}-\frac{d_{\mathrm{p}}}{v_{\mathrm{p}}}=2.00 \mathrm{~s}$

The horizontal distance travelled by the projectile is the same as the distance travelled by the sound.
$2.00 \mathrm{~s}=\frac{d_{\mathrm{p}}}{v_{\mathrm{s}}}-\frac{d_{\mathrm{p}}}{v_{\mathrm{p}}}$
$2.00 \mathrm{~s}=\frac{d_{\mathrm{p}}}{340 \mathrm{~m} / \mathrm{s}}-\frac{d_{\mathrm{p}}}{1250 \mathrm{~m} / \mathrm{s}}$
$2.00 \mathrm{~s}=\frac{(1250 \mathrm{~m} / \mathrm{s}) d_{\mathrm{p}}-(340 \mathrm{~m} / \mathrm{s}) d_{\mathrm{p}}}{(1250 \mathrm{~m} / \mathrm{s})(340 \mathrm{~m} / \mathrm{s})}$

$$
\begin{aligned}
2.00 \mathrm{~s} & =\frac{(910 \mathrm{~m} / \mathrm{s}) d_{\mathrm{p}}}{(1250 \mathrm{~m} / \mathrm{s})(340 \mathrm{~m} / \mathrm{s})} \\
d_{\mathrm{p}} & =\frac{(1250 \mathrm{~m} / \mathrm{s})(340 \mathrm{~m} / \mathrm{s})}{(910 \mathrm{~m} / \mathrm{s})} \times 2.00 \mathrm{~s} \\
d_{\mathrm{p}} & =934 \mathrm{~m}
\end{aligned}
$$

20. Five nodes implies four loops, or standing waves.

In turn, this implies the fourth harmonic.
$L=2 \lambda$

$$
\begin{aligned}
\lambda & =\frac{L}{2} \\
& =\frac{9.5 \mathrm{~m}}{2} \\
& =4.75 \mathrm{~m} \\
v & =\lambda f \\
& =(4.75 \mathrm{~m})(4.5 \mathrm{~Hz}) \\
& =21 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

21. $v=331 \mathrm{~m} / \mathrm{s}+\left(0.59 \mathrm{~m} / \mathrm{s} \cdot{ }^{\circ} \mathrm{C}\right)\left(23^{\circ} \mathrm{C}\right)$

$$
=344.6 \mathrm{~m} / \mathrm{s}
$$

$$
\lambda=\frac{v}{f}
$$

$$
=\frac{344.6 \mathrm{~m} / \mathrm{s}}{350 \mathrm{~Hz}}
$$

$$
=0.9846 \mathrm{~m}
$$

$$
L=\frac{\lambda}{4}
$$

$$
=0.246 \mathrm{~m}
$$

22. Calculate the fundamental frequency.

$$
\begin{aligned}
L & =\frac{\lambda}{4} \Rightarrow \lambda \\
& =4(1.35 \mathrm{~m}) \\
& =5.400 \mathrm{~m} \\
f & =\frac{v}{\lambda} \\
& =\frac{343 \mathrm{~m} / \mathrm{s}}{5.400 \mathrm{~m}} \\
& =63.5 \mathrm{~Hz}
\end{aligned}
$$

Calculate the frequency of the first harmonic.

$$
\begin{aligned}
L & =\frac{3 \lambda}{4} \Rightarrow \lambda \\
& =\frac{4(1.35 \mathrm{~m})}{3} \\
& =1.80 \mathrm{~m} \\
f & =\frac{v}{\lambda} \\
& =\frac{343 \mathrm{~m} / \mathrm{s}}{1.80 \mathrm{~m}} \\
& =191 \mathrm{~Hz}
\end{aligned}
$$

23. First, find the fundamental resonance.

$$
\begin{aligned}
L & =\frac{\lambda}{4} \\
& =\frac{145 \mathrm{~cm}}{4} \\
& =36.3 \mathrm{~cm}
\end{aligned}
$$

Next, find the second harmonic.

$$
\begin{aligned}
L & =\frac{3 \lambda}{4} \\
& =\frac{3(145 \mathrm{~cm})}{4} \\
& =109 \mathrm{~cm}
\end{aligned}
$$

Finally, calculate the third harmonic.

$$
\begin{aligned}
L & =\frac{5 \lambda}{4} \\
& =\frac{5(145 \mathrm{~cm})}{4} \\
& =182 \mathrm{~cm}
\end{aligned}
$$

$\mathbf{A}$ is the correct answer.
24. $L=\lambda$
$\lambda=1.35 \mathrm{~m}$
$f=\frac{v}{\lambda}$

$$
\begin{aligned}
& =\frac{343 \mathrm{~m} / \mathrm{s}}{1.35 \mathrm{~m}} \\
& =254 \mathrm{~Hz}
\end{aligned}
$$

25. The trumpet is a closed tube because it is closed at one end. An antinode always forms at the open end of a closed tube.

B is the correct answer.
26. The existence of four standing waves implies the fourth harmonic.

$$
\begin{aligned}
L & =2 \lambda \\
\lambda & =3.0 \mathrm{~m} \\
f & =\frac{10.0}{6.3 \mathrm{~s}} \\
& =1.59 \mathrm{~Hz} \\
v & =\lambda f \\
& =(3.0 \mathrm{~m})(1.59 \mathrm{~Hz}) \\
& =4.8 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

27. The frequency of the $n$th harmonic is equal to $n$ times the fundamental frequency.

$$
\begin{aligned}
n f_{\text {fund }} & =(3)(40.0 \mathrm{~Hz}) \\
& =1.20 \times 10^{2} \mathrm{~Hz}
\end{aligned}
$$

28. $L=\frac{\lambda}{2}$
$\lambda_{1}=2(0.25 \mathrm{~m})$

$$
=0.500 \mathrm{~m}
$$

$$
f_{1}=\frac{v}{\lambda_{1}}
$$

$$
=\frac{343 \mathrm{~m} / \mathrm{s}}{0.500 \mathrm{~m}}
$$

$$
=686.0 \mathrm{~Hz}
$$

$$
\lambda_{2}=2(0.28 \mathrm{~m})
$$

$$
=0.560 \mathrm{~m}
$$

$$
f_{2}=\frac{v}{\lambda_{2}}
$$

$$
=\frac{343 \mathrm{~m} / \mathrm{s}}{0.560 \mathrm{~m}}
$$

$$
=612.5 \mathrm{~Hz}
$$

$$
f_{\mathrm{b}}=f_{1}-f_{2}
$$

$$
=74 \mathrm{~Hz}
$$

29. $v=331 \mathrm{~m} / \mathrm{s}+\left(0.59 \mathrm{~m} / \mathrm{s} \cdot{ }^{\circ} \mathrm{C}\right)\left(28^{\circ} \mathrm{C}\right)$

$$
=347.5 \mathrm{~m} / \mathrm{s}
$$

$$
L=\frac{\lambda}{4}
$$

$$
\lambda=4(0.15 \mathrm{~m})
$$

$$
=0.600 \mathrm{~m}
$$

$$
f=\frac{v}{\lambda}
$$

$$
=\frac{347.5 \mathrm{~m} / \mathrm{s}}{0.600 \mathrm{~m}}
$$

$$
=5.8 \times 10^{2} \mathrm{~Hz}
$$

30. Blowing harder will result in a louder note because the amplitude of the sound wave will be larger.

C is the correct answer.
31. Since the man is moving away from the source, it takes longer for each compression to reach him. The frequency he hears will be lower than $f$.

D is the correct answer.
32. $v=\sqrt{\frac{(x T)}{m / L}} \Rightarrow v=\sqrt{x} \cdot \sqrt{\frac{T}{m / L}}$
$\Delta v \propto \sqrt{x}$
D is the correct answer.

